J
I
N
R

D
u
b
n
a

ON NEW GEOMETRICAL CONCEPT OF LOCAL QUANTUM FIELD

V.G. Kadyshevsky

Joint Institute for Nuclear Research
Dubna, Russia

The International Conference

“Gauge Field. Yesterday, Today, Tomorrow”

dedicated to Academician Andrey A.Slavnov`s

70th anniversary

Moscow, 20-23 January 2010

J I N R

D u b n a

The present talk is based on some ideas developed in the following papers:

1. Kadyshevsky V.G., Mateev M.D., Rodionov V.N., Sorin A.S., "TOWARDS A MAXIMAL MASS MODEL" arXiv:0708.4205v1 [hep-ph] 30 Aug 2007; CERN-TH/2007-150.
2. Kadyshevsky V.G., Nucl. Phys., B141, p. 477 (1978); in Proceedings of International Integrative Conference on Group Theory and Mathematical Physics, Austin, Texas, 1978; Fermilab-Pub.
78/70-THY, Sept. 1978; Phys. Elem. Chast. Atom. Yadra, 11, p. 5 (1980).
3. Kadyshevsky V.G., Mateev M. D., Phys. Lett., B106, p. 139 (1981); Nuovo Cimento. A87, p. 324, (1985).
4. Chizhov M. V., Donkov A.D., Ibadov R.M., Kadyshevsky V.G., Mateev M. D., Nuovo Cimento, A87, p. 350; p. 373 (1985).
5. Kadyshevsky V.G., Phys. Part. Nucl. 29, p. 227 (1998).

JINR Dubna

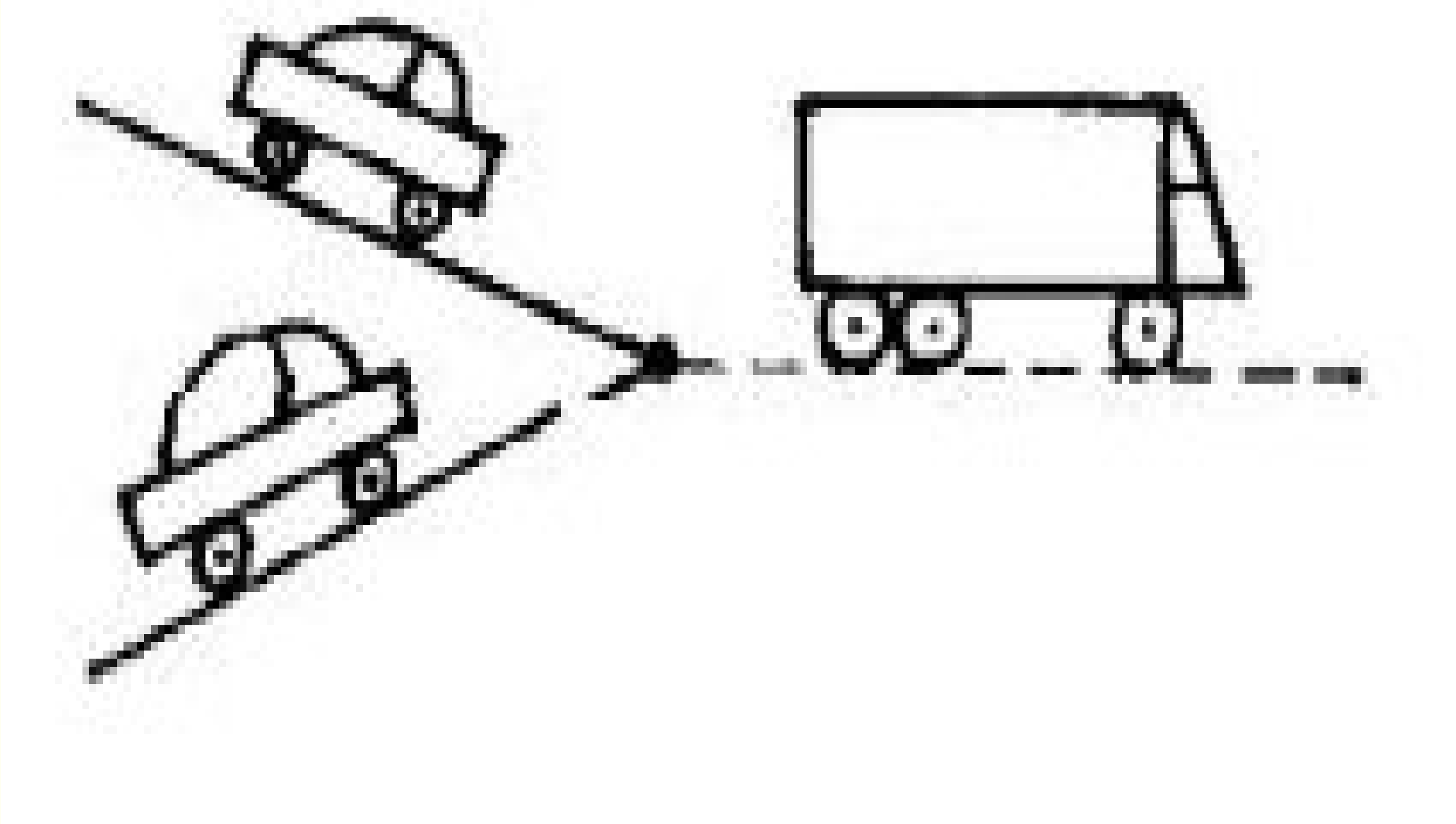
The contemporary theory of elementary particles is known as the Standard Model (SM). The notion 'elementary particle' supposes that in accordance with present experimental data these objects do not have a composite structure and are described by **local quantum fields**. The SM Lagrangian depends on finite numbers of fields of this kind:

- three families of quark and lepton fields;
- four vector boson fields $W^{\pm}, Z^{0}, \gamma$;
- an octet of gluon fields g;
- the hypothetic field of the Higgs boson H.

One of the most important characteristics of an elementary particle is its **mass**. In SM one observes a great variety in the mass values. For example, t-quark is more than 300000 times heavier than the electron. In this situation the question naturally arises: **up to what values of mass one may apply the concept of a local quantum field?** Formally, the contemporary QFT remains a logically perfect scheme and its mathematical structure does not change at all up to arbitrarily large values of masses of quanta.

J I N R
D u b n a

In other words, the ordinary QFT with its Feynman diagram techniques formally allows one to consider elementary processes for macrospically heavy objects.



Maybe this pathological picture is the Achilles heel of this theory?!

The key idea of our approach is the following radical hypothesis: the mass spectrum of elementary particles, i.e. the objects described by local fields, has to be cut off at a certain value M:

$$m \leq M \qquad (1)$$

This statement has to be accepted as a new fundamental principle of **Nature**, which similarly to the relativistic and quantum postulates should underlie QFT. The new universal physical constant M is not only the maximal value of particle mass but also plays the role of a new high-energy scale. We shall call this parameter the **fundamental mass**.

It is worth emphasizing that here, due to (1), the Compton wave length of a particle $\lambda_C = \hbar / mc$ cannot be smaller than the "*fundamental length*" $l = \hbar / Mc$. According to Newton T.D., Wigner E.P., Rev. Mod. Phys., 21, p.400 (1949), the parameter λ_C characterizes the dimensions of the region of space in which a relativistic particle of mass *m* can be localized. Therefore, the fundamental length l introduces into the theory a universal bound on the accuracy of the localization in space of elementary particles.

JINR Dubna

We start the construction of the new QFT based on condition (1) with the simplest example – the free theory of the neutral scalar field $\varphi(x)$:

$$(\Box + m^2)\varphi(x) = 0 \qquad (2)$$

It is the Klein-Gordon eqn. After standard Fourier transform

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int e^{-ip_\mu x^\mu} \varphi(p) d^4 p \quad (p_\mu x^\mu = p^0 x^0 - \vec{p}.x) \qquad (3)$$

we find the equation of motion in the Minkowski momentum 4-space: $(m^2 - p^2)\varphi(p) = 0, p^2 = p_0^2 - \vec{p}^{\,2}$

(4)

From a geometrical point of view m is the radius of the ”mass shell” hyperboloid: $m^2 = p_0^2 - \vec{p}^{\,2},$

(5)

J I N R

D u b n a

where the field φ(p) is defined. In the Minkowski momentum space one may embed hyperboloids of the type (5) of an **arbitrary** radius **m.**

How should one modify the equations of motion in order that the existence of the bound $m^2 \le M^2$ should become as evident as the limitation $v \le c$ in the special theory of relativity? In the latter case everything is explained in a simple way: the relativization of the 3-dimensional velocity space is equivalent to transition in this space from Euclidean to Lobachevsky geometry realized on the upper sheet of the 4-dimensional hyperboloid (5). Let us proceed in a similar way and substitute the 4-dimensional Minkowski momentum space, which is used in the standard QFT, by the (anti) de Sitter momentum space realized on the 5-hyperboloid:

$$p_0^2 - \vec{p}^{\,2} + p_5^2 = M^2 \qquad (6)$$

We suppose that in p-representation our scalar field is defined just on the surface (6), i.e. it is a function of five variables $(p_0, \vec{p}, p_5)$, which are connected by relation (6):

$$\delta(p_0^2 - \vec{p}^{\,2} + p_5^2 - M^2)\varphi(p_0, \vec{p}, p_5). \qquad (7)$$

The energy p_0 and the 3-momentum $\vec{p}$ here preserve their usual meaning and the mass shell relation (5) is satisfied as well. Therefore, for the field considered $\varphi(p_0, \vec{p}, p_5)$ the condition $m^2 \le M^2$ is always fulfilled. For this reason let us put $\frac{m}{M} \equiv \sin\mu$

Clearly, in eq. (7) the specification of a single function $\varphi(p_0, \vec{p}, p_5)$ of five variables (p_μ, p_5) is equivalent to the definition of two independent functions $\varphi_1(p)$ and $\varphi_2(p)$ of the 4-momentum p_μ :

$$\varphi(p_0, \vec{p}, p_5) \equiv \varphi(p, p_5) = \begin{pmatrix} \varphi(p, |p_5|) \\ \varphi(p, -|p_5|) \end{pmatrix} = \begin{pmatrix} \varphi_1(p) \\ \varphi_2(p) \end{pmatrix}, |p_5| = \sqrt{M^2 - p^2}. \qquad (8)$$

J I N R Dubna

The appearance of the new discrete degree of freedom $p_5/|p_5|$ and the associated doubling of the number of field variables is an **important feature of the new approach**. It must be taken into account in the search of the equation of motion for the free field in the (anti) de Sitter momentum space. Due to the mass shell relation (5) the Klein-Gordon equation (4) should also be satisfied by the field $\varphi(p_0, \vec{p}, p_5)$:

$$(m^2 - p_0^2 + \vec{p}^{\,2})\varphi(p_0, \vec{p}, p_5) = 0 \qquad (9)$$

From our point of view this equation is unsatisfactory for two reasons:

1. It does not reflect condition (1)
2. It cannot be used to determine the dependence of the field on the new quantum number $p_5/|p_5|$ in order to distinguish between the components $\varphi_1(p)$ and $\varphi_2(p)$

Here we notice that, because of (6), eq.(9) can be written as:

$$(p_5 + M\cos\mu)(p_5 - M\cos\mu)\varphi(p, p_5) = 0, \cos\mu = \sqrt{1 - \frac{m^2}{M^2}}. \qquad (10)$$

Now following the Dirac trick we postulate the equation of motion under question in the form:

$$2M(p_5 - M\cos\mu)\varphi(p, p_5) = 0 \tag{11}$$

Clearly, eq. (11) has none of the enumerated defects of the standard Klein-Gordon equation (9). However, equation (9) is still satisfied by the field $\varphi(p, p_5)$.

From eqs. (11) and (8) it follows that

$$2M(|p_5| - M\cos\mu)\varphi_1(p) = 0,$$
$$2M(|p_5| + M\cos\mu)\varphi_2(p) = 0, \tag{12}$$

and we obtain:

$$\varphi_1(p) = \delta(p^2 - m^2)\tilde{\varphi}_1(p) \tag{13}$$
$$\varphi_2(p) = 0$$

Therefore, the free field $\varphi(p, p_5)$ defined in the (anti)de Sitter momentum space (6) describes the same free scalar particles of mass m as the field φ*(p)* in the Minkowski p-space, with the only difference that now we necessarily have $m \le M$.

J I N R Dubna

The two-component structure (8) of the new field does not manifest itself on the mass shell, owing to (13). However, it will play an important role when the fields interact, i.e., off the mass shell.

Now we face the problem of constructing the action corresponding
to eq. (11) and transforming it to configuration representation.
Due to some reasons, not only technical, in the following we shall use the Euclidean formulation of the theory which appears as an analytical continuation to purely imaginary energies:

$$p_o \to ip_4 \quad (14)$$

In this case, instead of the (anti) de Sitter p-space (6) we shall work with de Sitter p-space:

$$-p_n^2 + p_5^2 = M^2, n = 1,2,3,4. \quad (15)$$

Obviously, $$p_5 = \pm\sqrt{M^2 + p^2}. \quad (16)$$

JINR Dubna

If one uses eq. (15), the Euclidean Klein-Gordon operator $(m^2 + p^2)$ may be written, similarly to (10), in the following factorized form:

$$m^2 + p^2 = (p_5 + M\cos\mu)(p_5 - M\cos\mu). \quad (17)$$

$$\cos\mu = \sqrt{1 - \frac{m^2}{M^2}}$$

Clearly, the nonnegative functional $S_0(M) = \pi M \times$

$$\int \frac{d^4 p}{|p_5|}\left[\varphi_1^+(p) 2M(|p_5| - M\cos\mu)\varphi_1(p) + \varphi_2^+(p) 2M(|p_5| + M\cos\mu)\varphi_2(p)\right], \quad (18)$$

$$\varphi_{1,2}(p) \equiv \varphi(p, \pm|p_5|), \quad (19)$$

plays the role of the action integral of the free Euclidean field $\varphi(p, p_5)$. The action may be written also as a 5 - integral:

$$S_0(M) = \pi M \times$$

$$\int \varepsilon(p_5)\delta(p_L p^L - M^2) d^5 p\left[\varphi^+(p, p_5) 2M(p_5 - M\cos\mu)\varphi(p, p_5)\right], \quad (20)$$

$$L = 1,2,3,4,5,$$

J I N R Dubna

where (21)

$$\varepsilon(p_5) = \frac{p_5}{|p_5|}.$$

What about the Fourier transform and the confuguration representation in the new formalism? Let us note that in the basic equation

$$-p_n^2 + p_5^2 = M^2$$

which defines the de Sitter p-space, all the components of the 5-momentum enter on equal footing. Therefore, the expression

$$\delta(p_L p^L - M^2)\varphi(p, p_5),$$

which now replaces (7), may be Fourier transformed in the following way:

$$\frac{2M}{(2\pi)^{3/2}} \int e^{-ip_K x^K} \delta(p_L p^L - M^2)\varphi(p, p_5) d^5 p = \varphi(x, x_5), \qquad (22)$$

$$K, L = 1,2,3,4,5.$$

J I N R Dubna

This function obviously satisfies the Klein-Gordon type equation in the *5-dimensional configuration space*:

$$\left(\frac{\partial^2}{\partial x_5^2} - \Box + M^2\right)\varphi(x, x_5) = 0 \qquad (23)$$

Integration over p_5 in (22) gives:

$$\varphi(x, x_5) = \frac{M}{(2\pi)^{3/2}} \int e^{ip_n x^n} \frac{d^4 p}{|p_5|} \left[e^{-i|p_5|x^5} \varphi_1(p) + e^{i|p_5|x^5} \varphi_2(p) \right], \qquad (24)$$

$$\varphi^+(x, x_5) = \varphi(x, -x_5), \qquad (25)$$

from which we get:

$$\frac{i}{M} \frac{\partial \varphi(x, x_5)}{\partial x_5} = \frac{1}{(2\pi)^{3/2}} \int e^{ip_n x^n} d^4 p \left[e^{-i|p_5|x^5} \varphi_1(p) - e^{i|p_5|x^5} \varphi_2(p) \right] \qquad (26)$$

J I N R Dubna

The four-dimensional integrals (24) and (26) transform the fields $\varphi_1(p)$ and $\varphi_2(p)$ to the configuration representation. The inverse transforms have the form:

$$\varphi_1(p) = \frac{-i}{2M(2\pi)^{5/2}} \int e^{-ip_n x^n} d^4x \left[\varphi(x,x_5) \frac{\partial e^{i|p_5|x^5}}{\partial x_5} - e^{i|p_5|x^5} \frac{\partial \varphi(x,x_5)}{\partial x_5} \right], \quad (27)$$

$$\varphi_2(p) = \frac{i}{2M(2\pi)^{5/2}} \int e^{-ip_n x^n} d^4x \left[\varphi(x,x_5) \frac{\partial e^{-i|p_5|x^5}}{\partial x_5} - e^{-i|p_5|x^5} \frac{\partial \varphi(x,x_5)}{\partial x_5} \right].$$

We note that the independent field variables

$$\varphi(x,0) \equiv \varphi(x) = \frac{M}{(2\pi)^{3/2}} \int e^{ip_n x^n} d^4p \frac{\varphi_1(p) + \varphi_2(p)}{|p_5|} \quad (28)$$

and

$$\frac{i}{M} \frac{\partial \varphi(x,0)}{\partial x_5} \equiv \chi(x) = \frac{1}{(2\pi)^{3/2}} \int e^{ip_n x^n} d^4p \left[\varphi_1(p) - \varphi_2(p) \right] \quad (29)$$

J I N R — Dubna

can be treated as initial Cauchy data on the surface $x_5 = 0$ for the hyperbolic-type equation (23).

Using(28) – (29) one can represent the action $S_0(M)$ in the following form:

$$S_0(M) = \frac{1}{2}\int d^4x\left[(\frac{\partial\varphi(x)}{\partial x_n})^2 + m^2(\varphi(x))^2 + M^2(\chi(x) - \cos\mu\varphi(x))^2 \right] \equiv$$

$$\doteq \int L_0(x,M)\,d^4x \qquad \textbf{(30)}$$

So we may conclude:

1.The new free Lagrangian density $L_0(x,M)$ is a **local function** of two field variables φ(x) and $\chi(x)$. This is a direct consequence of the fact that in the de Sitter momentum space the field has a doublet structure

$$\begin{pmatrix} \varphi_1(p) \\ \varphi_2(p) \end{pmatrix}$$

due to two signs of p_5 .

J I N R — Dubna

2. $L_0(x, M)$ does not contain a kinetic term corresponding to the field $\chi(x)$. Thus, this variable is just **auxiliary**.

3. Due to the presence of the factor $\cos\mu = \sqrt{1 - \frac{m^2}{M^2}}$ in the Lagrangian density $L_0(x, M)$ the condition (1) is fulfilled.

4. From the geometrical point of view the Euclidean momentum 4-space is the "flat limit" of the de Sitter p-space and may be associated with the aproximation

$$p_5 \cong M \qquad |p_n| \langle\langle M \tag{31}$$

In the same limit in configuration space we have:

$$\varphi(x, x_5) \cong e^{-iMx_5}\varphi(x) \tag{32a}$$

$$\chi(x) \cong \varphi(x) \tag{32b}$$

It is easy to obtain a correction of the order $0(\frac{1}{M^2})$ to the zero approximation (32b):

J I N R Dubna

$$\varphi(x) - \chi(x) \cong \frac{\Box\varphi(x)}{2M^2}$$

Thus, in the "flat limit" the quantity $L_0(x, M)$ coincides with the standard Euclidean Lagrangian density:

$$L_0(x, M) \cong \frac{1}{2}\left(\frac{\partial\varphi}{\partial x_n}\right)^2 + \frac{m^2\varphi^2(x)}{2}$$

("correspondence principle").

As an instructive example of interaction case let us consider the simplest scalar Lagrangian which is invariant with respect to the discrete symmetry transformation

$$\varphi(x) \to -\varphi(x), \chi(x) \to -\chi(x):$$

$$L(x, M) = \frac{1}{2}\left(\frac{\partial\varphi(x)}{\partial x_n}\right)^2 + \frac{1}{2}\left(M^2 - \frac{\lambda^2 v^2}{2}\right)(\varphi(x) - \chi(x))^2 + \qquad (33)$$

$$+ \frac{\lambda^2}{4}\left(\frac{\varphi^2(x) + \chi^2(x)}{2} - v^2\right)^2.$$

J I N R Dubna

under the condition $$M^2 - \frac{\lambda^2 v^2}{2} \geq 0$$

The spontaneouns symmetry breaking mechanism leads to the stable vacuum and appearance of particle mass:

$$m = m_0 \sqrt{1 - \frac{m_o^2}{4M^2}} \ , \qquad (34)$$

where $m_0 = \sqrt{2}\lambda v$. It is easy to see that

$$1 - \frac{m^2}{M^2} = \left(1 - \frac{m_0^2}{2M^2}\right)^2 \geq 0 \ ,$$

i.e. the condition (1) is fulfilled. If $M \to \infty$, then

$$L(x, M) \longrightarrow \frac{1}{2}\left(\frac{\partial \varphi}{\partial x_n}\right)^2 + \frac{\lambda^2}{4}\left(\varphi^2(x) - v^2\right)^2$$

and the particle mass is equal to $m = m_0 = \sqrt{2}\lambda v$

J I N R
D u b n a

Let us consider very briefly the new version of the free electromagnetic field theory. In the (anti) de Sitter p-space (6) we have the standard Maxwell equations for the 4-potential $A_\mu(p,p_5)$

$$p^2 A_\mu(p,p_5) = p_\mu(pA(p,p_5)), \qquad (35)$$

which are invariant under gauge transformations

$$A_\mu(p,p_5) \to A_\mu(p,p_5) - ip_\mu \lambda(p,p_5)$$

Let us put

$$(pA(p,p_5)) \equiv (M+p_5)A_5(p,p_5)$$

Then egn (35) can be written as

$$(M+p_5)\left[(M-p_5)A_\mu(p,p_5) - p_\mu A_5(p,p_5)\right] = 0$$

J I N R

D u b n a

Similarly to the scalar case (cf.(10) and (11)) we finally obtain the generalized Maxwell equations for the 5-potential

$(A_\mu(p,p_5), A_5(p,p_5))$ in the (anti) de Sitter momentum

space:

$$\begin{aligned} 2M(M-p_5)A_\mu(p,p_5) &= 2Mp_\mu A_5(p,p_5) \\ (M+p_5)A_5(p,p_5) &= (pA(p,p_5)) \end{aligned} \qquad (36)$$

The corresponding gauge transformations take the form:

$$\begin{aligned} A_\mu(p,p_5) &\to A_\mu(p,p_5) - ip_\mu\lambda(p,p_5) \\ A_5(p,p_5) &\to A_5(p,p_5) - i(M-p_5)\lambda(p,p_5) \end{aligned} \qquad (37)$$

It is clear that the standard Maxwell equations (35) follow from (36).

J I N R — D u b n a

The formulation of the given theory in configuration space will be developed using again, as in the scalar case, the de Sitter momentum space

$$-p_1^2 - p_2^2 - p_3^2 - p_4^2 + p_5^2 = M^2$$

The key role belongs to the 5-dimensional Fourier transform (compare with (22)):

$$A_L(x, x_5) = \frac{2M}{(2\pi)^{3/2}} \int e^{-ip_N x^N} \delta(p_K p^K - M^2) A_L(p, p_5) d^5 p, \quad (38)$$

$$K, L, N, = 1,2,3,4,5.$$

It is evident that (38) satisfies the equation (compare with (23))

$$\left(\frac{\partial^2}{\partial x_5^2} - \Box + M^2\right) A_L(x, x_5) = 0. \quad (39)$$

The action is given by the integrals (compare with (20), and (30))

$$S_0(M) = 2\pi M \times$$

$$\times \int \varepsilon(p_5)\delta(p_L p^L - M^2) d^5 p 2M(p_5 - M)\left| A_n(p,p_5) - \frac{p_n A_5(p,p_5)}{p_5 - M} \right|^2 =$$

$$= \int_{x_5=0} d^4 x L_0(x, M_5) = \frac{1}{4}\int_{x_5=0} d^4 x F^*_{KL}(x,x_5) F^{KL}(x,x_5) +$$

$$+ \frac{1}{2}\int_{x_5=0} d^4 x \left| \frac{\partial(e^{iMx_5} A_L(x,x_5))}{\partial x_L} - 2iMe^{iMx_5} A_5(x,x_5) \right|^2,$$

$n = 1,2,3,4;$ **K, L = 1, 2, 3, 4, 5,** **(40)**

where the "field strength 5-tensor":

$$F^{KL}(x,x_5) = \frac{\partial(e^{iMx_5} A_K(x,x_5))}{\partial x^L} - \frac{\partial(e^{iMx_5} A_L(x,x_5))}{\partial x^K} \quad \textbf{(41)}$$

is introduced. This quantity is obviously expressed in terms of the commutator of the 5-dimensional covariant derivatives:

$$D_L = \frac{\partial}{\partial x^L} - iqe^{iMx_5} A_L(x, x_5), \quad (42)$$

where q is the electric charge. It is easy to verify that the integral (40) is invariant under gauge transformations of the 5-potential $A_L(x, x_5)$ (compare with (37)):

$$e^{iMx_5} A_L(x, x_5) \longrightarrow e^{iMx_5} A_L(x, x_5) - \frac{\partial(e^{iMx_5}\lambda(x, x_5))}{\partial x^L} \quad (43)$$

with the condition

$$\left(\frac{\partial^2}{\partial x_5^2} - \Box + M^2\right)\lambda(x, x_5) = 0.$$

Let us notice that the gauge function $\lambda(x, x_5)$ is defined by two initial data λ(x) = λ (x,0) and $\mu(x) = \frac{i}{M}\frac{\partial \lambda(x,0)}{\partial x^5}$.

J I N R Dubna

Therefore, the new group (43) is broader than the standard gauge group. This is due to the fact that in the transition to the 5-dimensional description there appear additional superfluous gauge degrees of freedom, subject to removal.

To illustrate the technique developed, let us formulate in our terms a unique prescription for construction of the action integral of the Euclidean scalar electrodynamics consistent with the requirements of locality, gauge invariance, and de Sitter structure of momentum space:

1. In the action integral for the complex scalar field (use (30) as a pattern) it is necessary to substitute the ordinary derivatives (including $\frac{\partial}{\partial x^5}$ in $\chi(x) = \frac{i}{M} \frac{\partial}{\partial x_5} \varphi(x,0)$)

by the covariant ones (see (42)).

2. Add to the obtained expression the action integral of the electromagnetic field (40).

JINR Dubna

The total action integral remains invariant under simultaneous gauge transformation (43) at $x_5 = 0$ of the electromagnetic 5-potential and charged field transformation

$$\varphi(x) \to U(x,0)\varphi(x)$$
$$\chi(x) \to U(x,0)\left[\chi(x) + iq(\mu(x))\varphi(x)\right]$$

where

$$U(x,0) = U(x,x_5)\Big|_{x_5=0} = e^{iqe^{iMx_5}\lambda(x,x_5)}\Big|_{x_5=0} = e^{iq\lambda(x)},$$

$$\lambda(x) = \lambda(x,0), \mu(x) = \frac{i}{M}\frac{\partial\lambda(x,0)}{\partial x_5}$$

Let us note that a generalization of the considered Abelian formalism to the non- Abelian case does not meet any difficulties.

In conclusion, I would like to discuss briefly some peculiar features of the new version of the Euclidean fermion theory. In the ordinary formalism the free Euclidean Dirac operator

$$D(p) = m + p_n\gamma^n ; n = 1,2,3,4 \qquad (44)$$

appears as a result of factorization of the Euclidean K.-G.wave operator:

$$p_n^2 + m^2 = (m + p_n\gamma^n)(m - p_n\gamma^n) \qquad (45)$$

Now, instead of (45) we obtain the following factorization formula:

$$2M(p_5 - M\cos\mu) = \left[2M\sin\mu/2 + p_n\gamma^n - (p_5 - M)\gamma^5\right]\left[2M\sin\mu/2 - p_n\gamma^n + (p_5 - M)\gamma^5\right] \qquad (46)$$

and, correspondingly, instead of (44) the new expression for the Dirac operator

$$D(p,M) = p_n\gamma^n - (p_5 - M)\gamma^5 + 2M\sin\mu/2 \qquad (47)$$

It is easy to check that in the "flat approximation"

$$|p_n| << \mathrm{M},\ \mathrm{m} << \mathrm{M} \qquad , \qquad p_5 \simeq \mathrm{M}$$

JINR Dubna

both expressions (47) and (44) coincide. The operator (47) allows us to develop the **local** spinor field formalism in configuration space that can be considered as a generalization of the Euclidean Dirac theory along our lines

But the amusing point is that the new KG-operator

$2M(p_5 - M\cos\mu)$ has one more decomposition into matrix factors:

$$2M(p_5 - M\cos\mu) =$$

$$= \left[p_n\gamma^n - \gamma^5(p_5 + M) + 2M\cos{\mu/2} \right]\left[p_n\gamma^n - \gamma^5(p^5 + M) - 2M\cos{\mu/2} \right] \qquad (48)$$

Therefore, if our approach is considered to be realistic, it may be assumed that in Nature there exists some **exotic fermion field** associated with the wave operator

$$D_{exotic}(p, M) = p_n\gamma^n - \gamma^5(p_5 + M) + 2M\cos{\mu/2} \qquad (49)$$

In contrast to $D(p,M) = p_n\gamma^n - (p_5 - M)\gamma^5 + 2M\sin{\mu/2}$, the operator $D_{exotic}(p,M)$ does not have a limit when $M \to \infty$, that justifies the name chosen for the field considered. The polarization properties of the exotic fermion field differ sharply from the standard ones.

It is tempting to think that the quanta of the exotic fermion field have a direct relation to the structure of the "dark matter"

Let us come back to the de Sitter surface

$$-p_n^2 + p_5^2 = M^2 \qquad (50)$$

Using the matrix basis $(\gamma^1,\gamma^2,\gamma^3,\gamma^4,\gamma^5)$ one may represent (50)

as $(M + p_K\Gamma^L)(M - p_K\Gamma^K) = M^2 - p_K p^K = 0,$ K =1,2,3,4,5

J I N R Dubna

For spinor fields, which are defined on the surface (50), the matrix operators

$$\frac{1}{2M}(M + p_K\gamma^K) \equiv \Pi_R(p, p_5)$$
$$\frac{1}{2M}(M - p_K\gamma^K) \equiv \Pi_L(p, p_5) \qquad \textbf{(51)}$$

are projection operators. In other words,

$$\Pi_R^2 = \Pi_R, \Pi_L^2 = \Pi_L \qquad \textbf{(52)}$$
$$\Pi_R\Pi_L = \Pi_L\Pi_R = 0$$
$$\Pi_R + \Pi_L = 1$$

So the fermion field $\psi(p, p_5)$, defined in the de Sitter momentum space, may be presented as a sum of two fields

$$\psi(p, p_5) = \psi_R(p, p_5) + \psi_L(p, p_5) \qquad \textbf{(53)}$$

J I N R Dubna

$$\psi_R(p,p_5) = \Pi_R \psi(p,p_5), \quad \psi_L(p,p_5) = \Pi_L \psi(p,p_5)$$

which obey the following 5-dimensional Dirac equations:

$$(M - p_K \gamma^K)\psi_R(p,p_5) = 0$$
$$(M + p_K \gamma^K)\psi_L(p,p_5) = 0 \qquad (54)$$

Obviously, decomposition (53) is de Sitter invariant. It is easy to verify that in the "flat approximation" $|p_n| << M, \quad \simeq M p_5$

one has

$$\Pi_{R,L} = \frac{1 \pm \gamma^5}{2} \qquad (55)$$

This is the reason that we can consider the fields $\psi_R(p, p_5)$ and $\psi_L(p, p_5)$ as "chiral" components in our approach. The new chirality operator $\frac{p_L \gamma^L}{M}$, similarly to its "flat counterpart", has eigenvalues equal to ± 1 but depends on the energy and momentum.

J I N R — Dubna

It is well khown that the chiral fermions are the basic spinor field variables in SM. The new geometrical nature of these quantities has to manifest itself at high energies E ≥ M.

In configuration space the 5-dimensional Dirac equations (54)take the form

$$\left[M - i\frac{\partial}{\partial x^K}\gamma^K\right]\psi_R(x, x_5) = 0$$

$$\left[M + i\frac{\partial}{\partial x^K}\gamma^K\right]\psi_L(x, x_5) = 0 \qquad (56)$$

$$K = 1,2,3,4,5$$

Introducing the corresponding initial conditions at $x_5 = 0$

$$\psi_R(x,0) \equiv \psi_{(R)}(x)$$

$$\psi_L(x,0) \equiv \psi_{(L)}(x)$$

one obtains the local fields which can undergo chiral gauge transformations.

The new geometrical concept of chirality allows us to think that the parity violation in weak interactions discovered more than 50 years ago was a manifestation of the de Sitter nature of momentum 4-space.

The main purpose of the present talk was to demonstrate that there exist a local field formalism respecting the gauge invariance principle and being consistent with our main hypothesis $m \leq M$.

A nontrivial generalization of the Standard Model based on our geometrical approach, in particular, on a new concept of chirality, now is being worked out. Below we give, just for illustation the new interaction Lagrangian expession for leptons e and ν_e containing corrections of the order 0 (1/M).

$$\mathcal{L}_{int} = e\,\bar{e}(x)\not{A}e(x) - \bar{g}\sin^2\theta_w \bar{e}(x)\not{Z}e(x) - \frac{\bar{g}}{2}\bar{\nu}(x)\not{Z}\frac{1-\gamma^5}{2}\nu(x) -$$

$$-\frac{g}{\sqrt{2}}\bar{\nu}(x)\not{W}\frac{1-\gamma^5}{2}e(x) - \frac{g}{\sqrt{2}}\bar{e}(x)\not{W}\frac{1-\gamma^5}{2}\nu(x) + \frac{\bar{g}}{2}\bar{e}(x)\not{Z}\frac{1-\gamma^5}{2}e(x) +$$

$$+\frac{ig}{4\sqrt{2}M}[\bar{\nu}(x)\not{W}^+\overrightarrow{\not{\partial}}e(x) - \bar{e}(x)\overleftarrow{\not{\partial}}\not{W}\nu(x)] + \frac{ig}{4\sqrt{2}M}[\bar{e}(x)\not{W}\overrightarrow{\not{\partial}}\nu(x) - \bar{\nu}(x)\overleftarrow{\not{\partial}}\not{W}^+ e(x)] +$$

$$+\frac{i\bar{g}}{8M}[\bar{e}(x)\not{Z}\overrightarrow{\not{\partial}}e(x) - \bar{e}(x)\overleftarrow{\not{\partial}}\not{Z}e(x)] + \frac{i\bar{g}}{8M}[\bar{\nu}(x)\not{Z}\overrightarrow{\not{\partial}}\nu(x) - \bar{\nu}(x)\overleftarrow{\not{\partial}}\not{Z}\nu(x)] +$$

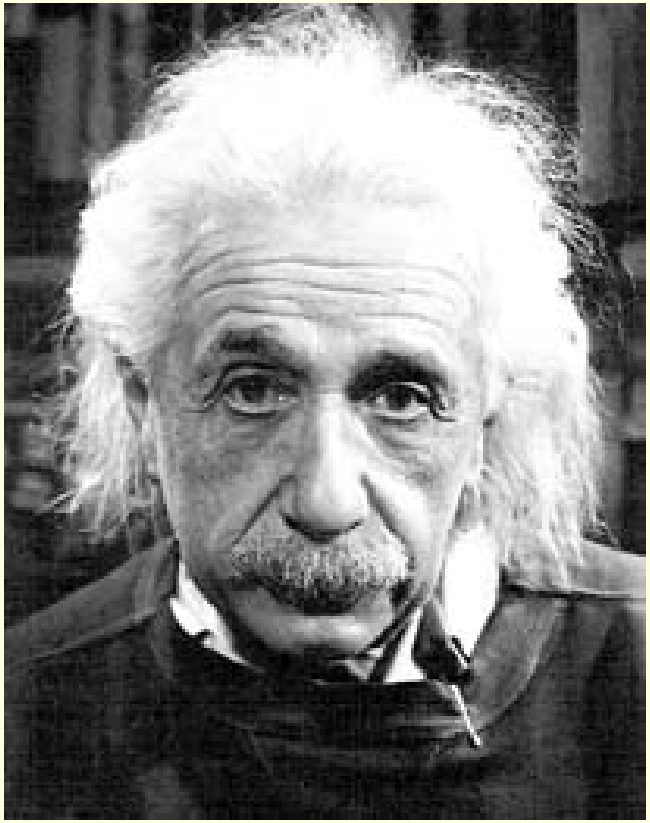
$$+\frac{H(x)}{v}\left\{m_\nu\bar{\nu}(x)\nu(x) + \frac{i\bar{g}}{8M}[\bar{\nu}(x)\not{Z}\overrightarrow{\not{\partial}}\nu(x) - \bar{\nu}(x)\overleftarrow{\not{\partial}}\not{Z}\nu(x)]\right\} +$$

$$+\frac{H(x)}{v}\left\{m_e\bar{e}(x)e(x) + \frac{i\bar{g}}{8M}[\bar{e}(x)\not{Z}\overrightarrow{\not{\partial}}e(x) - \bar{e}(x)\overleftarrow{\not{\partial}}\not{Z}e(x)]\right\} +$$

$$+\frac{ig}{4\sqrt{2}M}\frac{H(x)}{v}\left\{[\bar{\nu}(x)\not{W}^+\overrightarrow{\not{\partial}}e(x) - \bar{e}(x)\overleftarrow{\not{\partial}}\not{W}\nu(x)] + [\bar{e}(x)\not{W}^+\overrightarrow{\not{\partial}}\nu(x) - \bar{\nu}(x)\overleftarrow{\not{\partial}}\not{W}e(x)]\right\}$$

All fields are defined in the Minkowski x-space. According to our estimaions M $\gtrsim$ *1TeV*

INSTEAD OF EPILOGUE



"EXPERIMENT =
GEOMETRY + PHYSICS"