The puzzling infrared of QCD: the Landau gauge case

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Outline of the talk

1. Introduction, motivation: the infrared QCD debate
2. How to compute Landau gauge gluon and ghost propagators on the lattice
3. (Preliminary) results for gluon, ghost propagators and running coupling in lattice quenched and full QCD
4. Gribov copies, continuum limit, finite-volume effects, multiplicative renormalization as seen in $SU(2)$ lattice gauge theory
5. Conclusion and outlook

1. Introduction, motivation

Landau gauge gluon, ghost, quark propagators and vertex functions:

$\Rightarrow$ allow to fix phenomenologically useful parameters:
effective (dynamical) gluon mass m_g, Λ_{QCD}, $\langle\overline{\psi}\psi\rangle$, $\langle A^2\rangle$ (?), ...;

$\Rightarrow$ can be directly used as input for hadron phenomenology:
Bethe-Salpeter eqs. for mesons, Faddeev eqs. for baryons,
cf. Alkofer, Eichmann, Krassnigg, Nicmorus, arXiv:0912.3105 [hep-ph];

$\Rightarrow$ their infrared behaviour is related to confinement criteria:
Gribov-Zwanziger, Kugo-Ojima, violation of positivity,...;

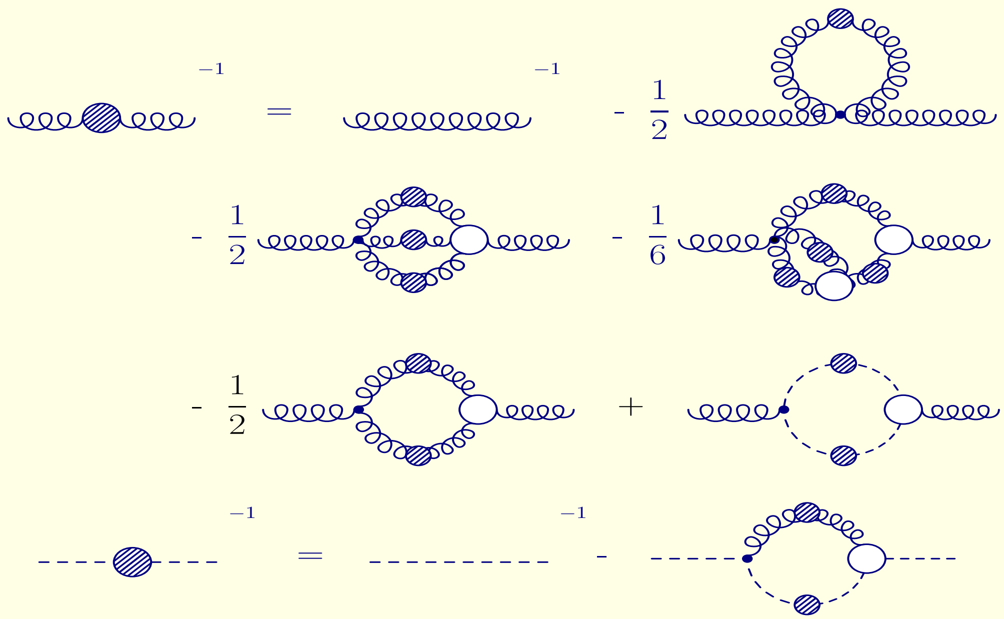
$\Rightarrow$ for $T > 0$ allow for determining screening length
and other characteristica at T_c.

$\Longrightarrow$ Intensive non-perturbative investigations in the continuum and on the lattice over many years.

$\Longrightarrow$ Infrared (IR) limit of special interest.

Landau gauge Green's functions in the continuum determined from (truncated) Dyson-Schwinger (DS) and Wetterich funct. RG (FRG) eqs. taking into account Slavnov-Taylor identities (STI)

[Alkofer, Aguilar, Boucaud, Dudal, Fischer, Pawlowski, von Smekal, Zwanziger,.. ('97 - '09)]



$$\Rightarrow \quad D^{ab}_{\mu\nu} = \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\frac{Z(q^2)}{q^2}$$

$$\Rightarrow \quad G^{ab} = \delta^{ab}\,\frac{J(q^2)}{q^2}$$

Running coupling from ghost-ghost-gluon vertex in MOM scheme assuming $\tilde{Z}_1 \equiv 1$:

$$\alpha_s(q^2) = \frac{g_0^2}{4\pi}\; Z(q^2)\cdot[J(q^2)]^2$$

Infrared "scaling" solution of DS and FRG eqs.

[Alkofer, Fischer, Lerche, Maas, Pawlowski, von Smekal, Zwanziger,... ('97 - '09)]

$$Z(q^2) \propto (q^2)^{\,\kappa_D}\,, \quad J(q^2) \propto (q^2)^{-\kappa_G} \quad \text{for} \quad q^2 \to 0$$

with

$$\kappa_D = 2\,\kappa_G + (4-d)/2 \quad \Longrightarrow \quad \kappa_D = 2\,\kappa_G, \quad \kappa_G \simeq 0.59 \quad \text{for} \quad d = 4$$

is claimed

- to be consistent with BRST quantization,
- to hold without any truncation of the tower of DS or FRG eqs.,
- to be independent of the number of colors N_c,
- to look qualitatively the same in any dimension $d = 2, 3, 4$.

Running coupling:

$$\alpha_s(q^2) \to \text{const.} \quad \text{for} \quad q^2 \to 0$$

i.e. infrared fixed point as in analytic perturbation theory

[D.V. Shirkov, I.L. Solovtsov ('97 - '02)].

Alternative "decoupling" IR solution

[Boucaud et al. ('06 -'08), Aguilar et al. ('07-'08), Dudal et al. ('05-'08)]

$$\kappa_D = 1\,, \;\; \kappa_G = 0$$

i.e.

$$D(q^2) = Z(q^2)/q^2 \to \text{const.}\,, \qquad J(q^2) \to \text{const.}$$

such that

$$\alpha_s(q^2) = \frac{g_0^2}{4\pi}\, Z(q^2) \cdot [J(q^2)]^2 \quad \to 0 \;\; \text{for} \;\; q^2 \to 0\,.$$

Existence has been confirmed.

[Fischer, Maas, Pawlowski, Annals Phys. '09, arXiv:0810-1987 [hep-ph]]

No debate any more on who is right, but about criteria what is the physically correct solution (BRST).

Claim: $J(0)$ to be chosen as an IR boundary condition.

Expect: close relation to the notorious Gribov problem.

Question: Relevance for phenomenology ?

IR "scaling" solution for Z, J at a first view in agreement with confinement scenarios:

- Kugo-Ojima confinement criterion [Ojima, Kugo ('78 - '79)]:
  absence of colored physical states $\Longleftrightarrow$ ghost (gluon) propagator more (less) singular than simple pole for $q^2 \to 0$.

- Gribov-Zwanziger confinement scenario
  [Gribov ('78), Zwanziger ('89 - ...)]:
  gauge fields restricted the Gribov region

$$\Omega = \Big\{ A_\mu(x) : \ \partial_\mu A_\mu = 0, \ M_{FP} \equiv -\partial D(A) \geq 0 \Big\}$$

  are accumulated at the Gribov horizon $\partial\Omega$:

  non-trivial eigenvalues of M_{FP}: $\lambda_0 \to 0$.

$$\Longrightarrow \quad \begin{array}{ll} \text{Ghost:} & J(q^2) \ \to \ \infty \\ \text{Gluon:} & D(q^2) \ \to \ 0 \end{array} \qquad \text{for} \ \ q^2 \to 0.$$

There are attempts to modify scenarios such, that IR "decoupling" solution can be accomodated, too. [Dudal et al. ('08 - '09), Kondo ('09)].

The Gribov problem:

- Existence of many gauge copies inside Ω.
- What are the right copies?
  Restriction inside Ω to fundamental modular region (FMR) required?

$$\Lambda = \left\{ A_\mu(x) : \; F(A^g) < F(A) \;\; \text{for all} \;\; g \neq \mathbf{1} \right\} .$$

Answer in the limit of infinite volume [Zwanziger ('04)]:

Non-perturbative quantization requires only restriction to Ω,

$$\text{i.e.} \quad \delta_\Omega(\partial_\mu A_\mu)\, \det(-\partial_\mu D_\mu^{ab}) e^{-S_{YM}[A]} .$$

Expectation values taken on Ω or Λ should be equal in the thermodynamic limit.

- What happens on a (finite) torus?
- How Gribov copies influence finite-size effects?

Questions to Yang-Mills theory on the lattice:

- What kind of infrared DS and FRG solutions are supported ?
- What is the influence of Gribov copies on gluon and ghost propagators ?
- Finite-volume effects ?
- Continuum limit, scaling, non-perturbative multiplicative renormalization at finite volume ?

Lattice investigations of gluon and ghost propagators carried out over many years:

Adelaide: Bonnet, Leinweber, Williams, et al.;

Berlin: Burgio, Ilgenfritz, M.-P., Sternbeck, et al.;

Dubna/Protvino: Bakeev, Bogolubsky, Bornyakov, Mitrjushkin;

San Carlos: Cucchieri, Maas, Mendes;

Paris: Boucaud, Leroy, Pene, et al.;

Coimbra: Oliveira, Silva;

Tübingen: Bloch, Langfeld, Reinhardt, Watson et al.;

Utsunomiya: Furui, Nakajima.

2. How to compute Landau gauge gluon and ghost propagators on the lattice

A few technicalities:

i) Generate lattice discretized gauge fields $U = \{U_{x,\mu} \in SU(N_c)\}$ by MC simulation from path integral

$$Z_{\text{Latt}} = \int \prod_{x,\mu} [dU_{x,\mu}] \; (\det\, Q(\kappa, U))^{N_f} \;\; \exp(-S_G(U))$$

– standard Wilson plaquette action

$$S_G(U) \;\; = \;\; \beta \sum_x \sum_{\mu<\nu} \left(1 - \frac{1}{N_c} \,\mathfrak{Re}\, \mathrm{Tr}\, U_{x,\mu\nu} \right),$$

$$U_{x,\mu\nu} \;\; \equiv \;\; U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}, \qquad \beta \equiv 2N_c/g_0^2$$

– (clover-improved) Dirac-Wilson fermion operator $Q(\kappa, U)$:

$N_f = 0$ – pure gauge case,

$N_f = 2$ – full QCD with equal bare quark masses $\quad ma = 1/2\kappa - 1/2\kappa_c$,

$a(\beta)$ – lattice spacing.

ii) Z_{Latt} is simulated with (Hybrid) MC method without gauge fixing.

iii) Gauge fix each lattice field U:

$$U^g_{x\mu} = g_x \cdot U_{x\mu} \cdot g^\dagger_{x+\hat{\mu}}$$

standard orbits: $\{g_x\}$ periodic on the lattice;

extended orbits: $\{g_x\}$ periodic up to global $Z(N)$ transformations;

Landau gauge: $\mathcal{A}_{x+\hat{\mu}/2,\mu} = \frac{1}{2iag_0}\left(U_{x\mu} - U^\dagger_{x\mu}\right)\Big|_{\text{traceless}}$

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4}\left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) = 0$$

equivalent to maximizing the gauge functional

$$F_U(g) = \sum_{x,\mu} \frac{1}{N_c}\,\mathfrak{Re}\ \mathrm{Tr}\, U^g_{x\mu} = \mathrm{Max.}\,.$$

For uniqueness (FMR) one requires to find the global maximum

[Parrinello, Jona-Lasinio ('90), Zwanziger ('90)].

Well understood in compact $U(1)$ theory in order to get

e.g. massless photon propagator

[Bogolubsky, Bornyakov, Mitrjushkin, M.-P., Peters, Zverev ('93 - '99)].

Optimized maximization in (our) practice: simulated annealing (SA) + overrelaxation (OR)

Gribov problem: global maximum of $F_U(g)$ very hard or impossible to find.

"Best copy strategy": repeated initial random gauges

$\Longrightarrow$ best copies (bc) from subsequent SA + OR maximizations,

$\Longrightarrow$ compared with first (random) copies (fc)).

iv) Compute propagators

- Gluon propagator:

$$D^{ab}_{\mu\nu}(q) = \left\langle \widetilde{A}^a_\mu(k)\widetilde{A}^b_\nu(-k)\right\rangle \equiv \delta^{ab}\left(\delta_{\mu\nu} - \frac{q_\mu\ q_\nu}{q^2}\right) D(q^2)$$

for lattice momenta

$$q_\mu(k_\mu) = \frac{2}{a}\sin\left(\frac{\pi k_\mu}{L_\mu}\right), \qquad k_\mu \in \left(-L_\mu/2, L_\mu/2\right]$$

with certain cuts in order to suppress artifacts of lattice discretization.

- Ghost propagator:

$$G^{ab}(q) = \frac{1}{V^{(4)}} \sum_{x,y} \left\langle \mathrm{e}^{-2\pi i\, k\cdot(x-y)} [M^{-1}]^{ab}_{xy} \right\rangle \equiv \delta^{ab} G(q)\,.$$

$M \sim \partial_\mu D_\mu$ - Landau gauge Faddeev-Popov operator

$$M^{ab}_{xy}(U) = \sum_\mu A^{ab}_{x,\mu}(U)\,\delta_{x,y} - B^{ab}_{x,\mu}(U)\,\delta_{x+\hat{\mu},y} - C^{ab}_{x,\mu}(U)\,\delta_{x-\hat{\mu},y}$$

$$\begin{aligned} A^{ab}_{x,\mu} &= \Re\mathrm{e}\,\mathrm{Tr}\left[\{T^a, T^b\}(U_{x,\mu} + U_{x-\hat{\mu},\mu})\right], \\ B^{ab}_{x,\mu} &= 2\cdot\Re\mathrm{e}\,\mathrm{Tr}\left[T^b T^a\, U_{x,\mu}\right], \\ C^{ab}_{x,\mu} &= 2\cdot\Re\mathrm{e}\,\mathrm{Tr}\left[T^a T^b\, U_{x-\hat{\mu},\mu}\right], \qquad \mathrm{Tr}\,[T^a T^b] = \delta^{ab}/2\,. \end{aligned}$$

M^{-1} from solving

$$M^{ab}_{xy}\phi^b(y) = \psi^a_c(x) \equiv \delta^{ac} \exp(2\pi i\, k\cdot x)$$

with (preconditioned) conjugate gradient algorithm.

3. (Preliminary) results for gluon, ghost propagators, and running coupling in lattice quenched and full QCD

- Pure gauge case $N_f = 0$:
  $\beta = 5.7, 5.8, 6.0, 6.2$; $12^4, \ldots, 56^4$, $aL_{max} \simeq 9.5$fm;
  $\beta = 5.7$; $64^4, \ldots, 96^4$, $aL_{max} \simeq 16.3$fm.

- Full QCD case $N_f = 2$:
  thanks: configurations provided by QCDSF - collaboration,
  $\beta = 5.29, 5.25$; mass parameter $\kappa = 0.135, ..., 0.13575$;
  $16^3 \times 32$, $24^3 \times 48$.

- We have computed propagators or dressing functions

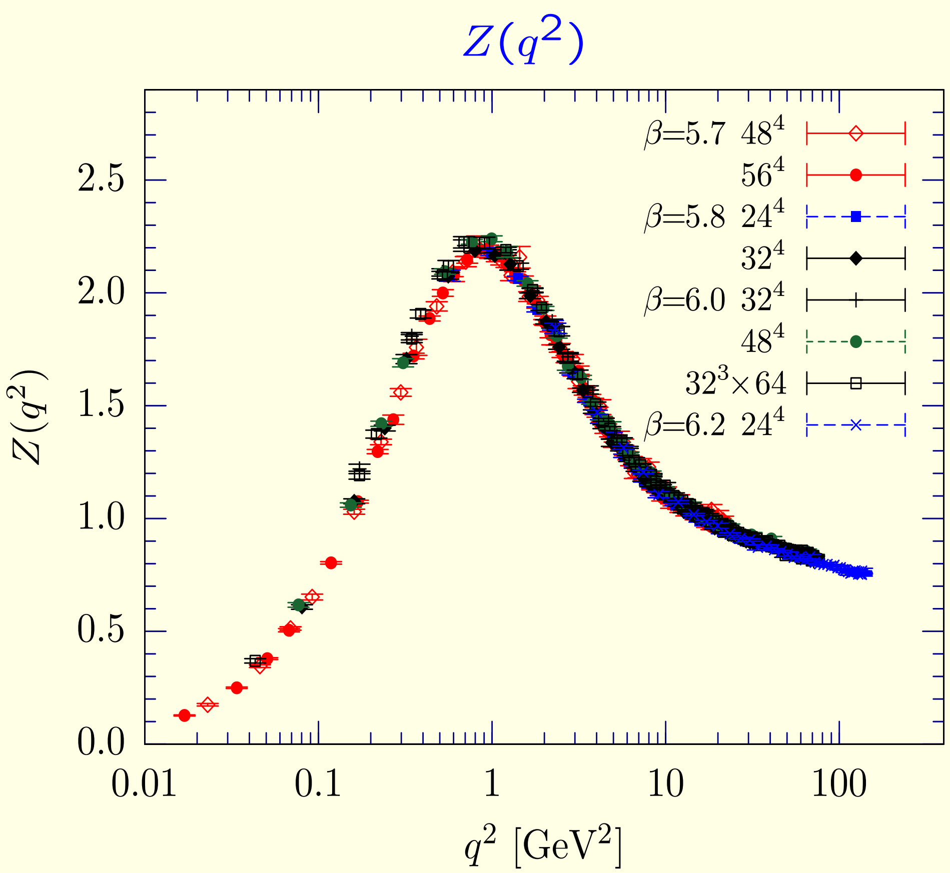
$$\text{Gluon} \quad Z(q^2) \equiv q^2 D(q^2), \qquad \text{Ghost} \quad J(q^2) \equiv q^2 G(q^2)$$

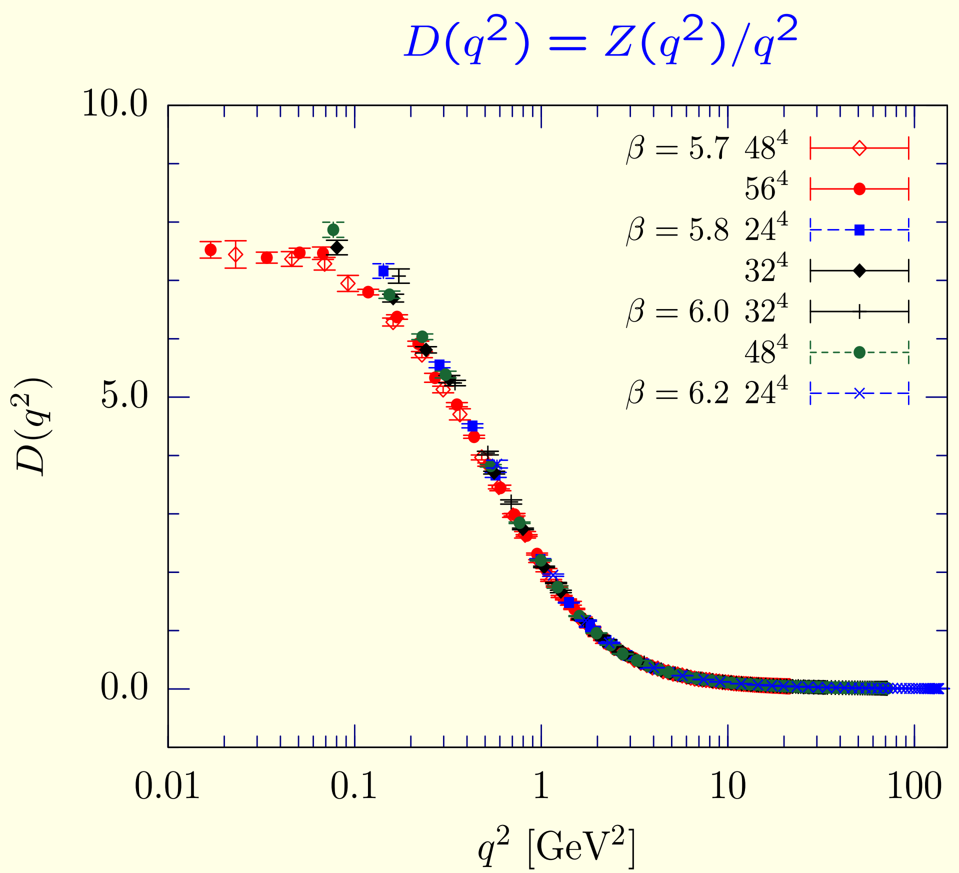
  as well as ghost-ghost-gluon vertex and Kugo-Ojima parameter.

Gluon dressing function and propagator from first OR copies

quenched QCD ($N_f = 0$), renorm. point: $q = \mu = 4\text{GeV}$

[Sternbeck, Ilgenfritz, M.-P., Schiller, PRD 72 (2006), Proc. IRQCD '06]





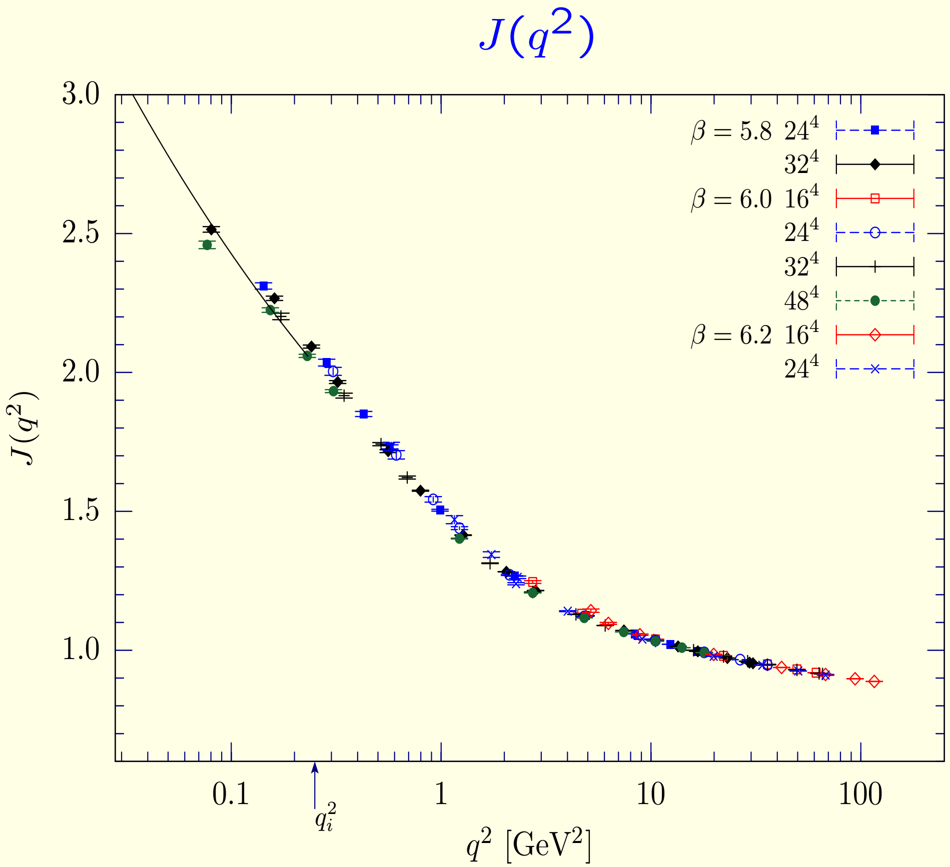
$\Longrightarrow$ $D(q^2) = Z(q^2)/q^2$ shows plateau and not $D(q^2) \to 0$ for $q^2 \to 0$,

$\Longrightarrow$ corresponds to an effective gluon mass.

Ghost dressing function from first OR copies

quenched QCD ($N_f = 0$), renorm. point: $q = \mu = 4\text{GeV}$

[Sternbeck, Ilgenfritz, M.-P., Schiller, PRD 72 (2006), Proc. IRQCD '06]



$\Longrightarrow$ looks still singular but with too small exponent

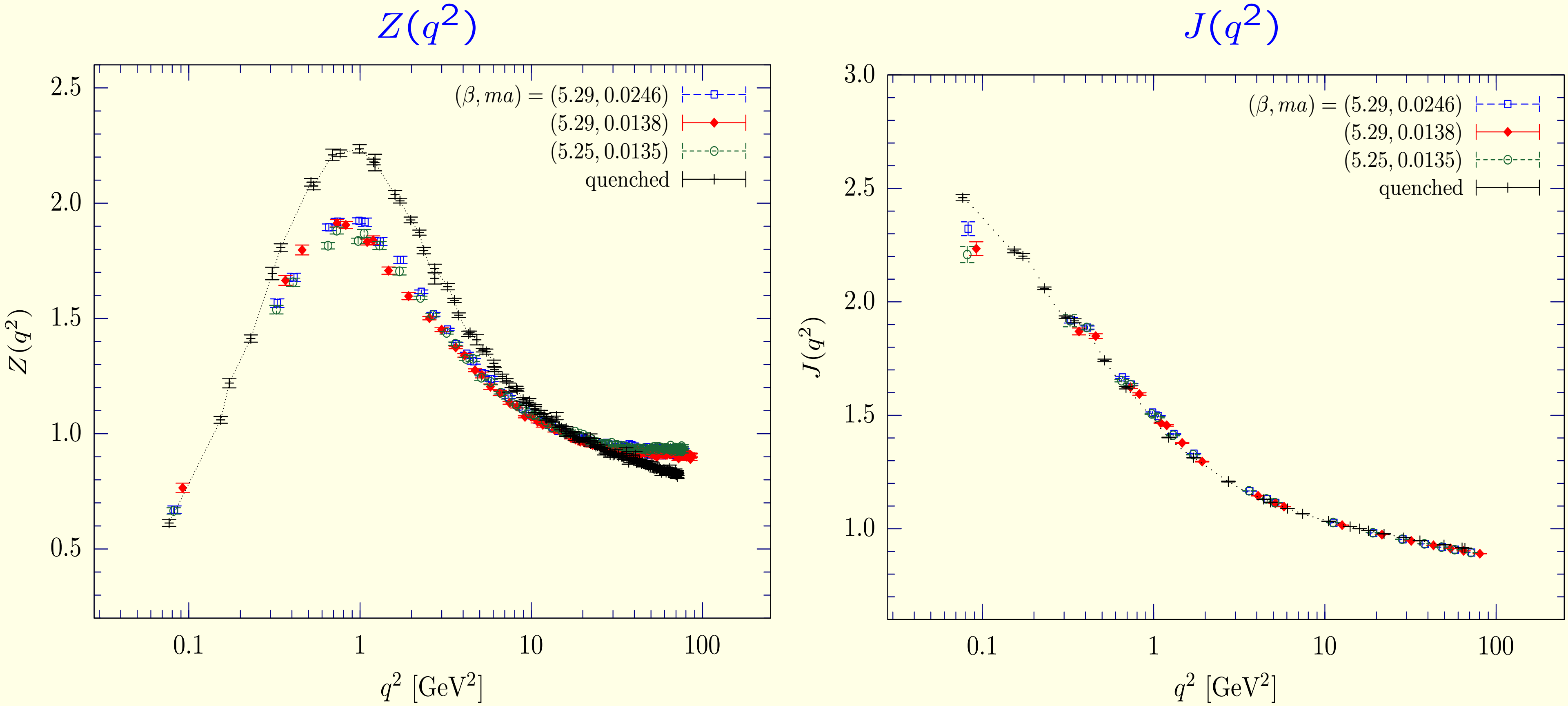
in comparison with "scaling" solution;

$\Longrightarrow$ no serious finite-volume effects (??).

Gluon and ghost dressing functions from first OR copies

full QCD ($N_f = 2$) versus quenched QCD ($N_f = 0$), renorm. point: $q = \mu = 4$GeV

[Ilgenfritz, M.-P., Schiller, Sternbeck (A. DiGiacomo 70, '06)]



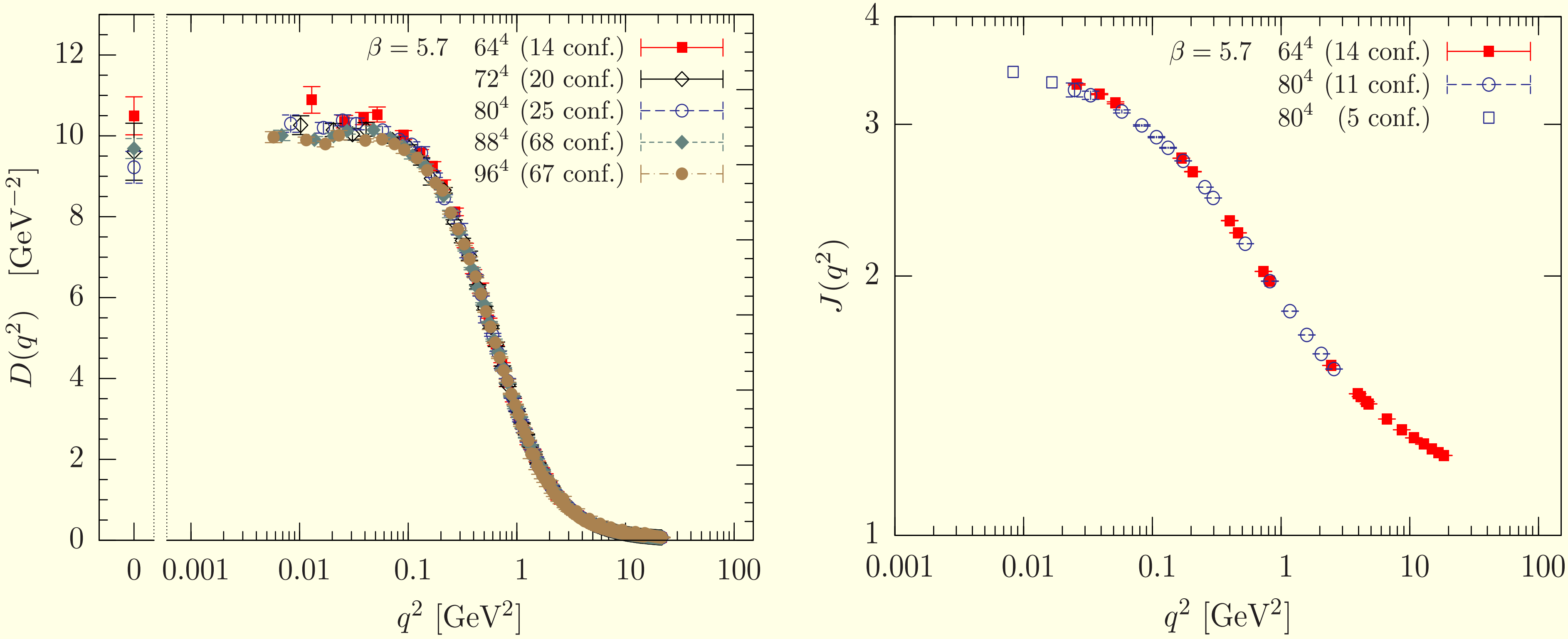
$\Longrightarrow$ Influence of virtual quark loops in $Z(q^2)$ clearly visible.

$\Longrightarrow$ No quenching effect in $J(q^2)$, as ghosts do not directly couple to quarks.

Gluon propagator and ghost dressing function on large volumes

quenched QCD, first but long run SA + OR copies, unrenormalized

[Bogolubsky, Ilgenfritz, M.-P., Sternbeck, PLB 676 (2009)]



$\Longrightarrow$ $D(q^2)$ still shows plateau behaviour, $J(q^2)$ seems to tend to const.,

$\Longrightarrow$ clear indication for "decoupling" solution.

$\Longrightarrow$ However, we used strong coupling, i.e. coarse lattices. Continuum limit ?

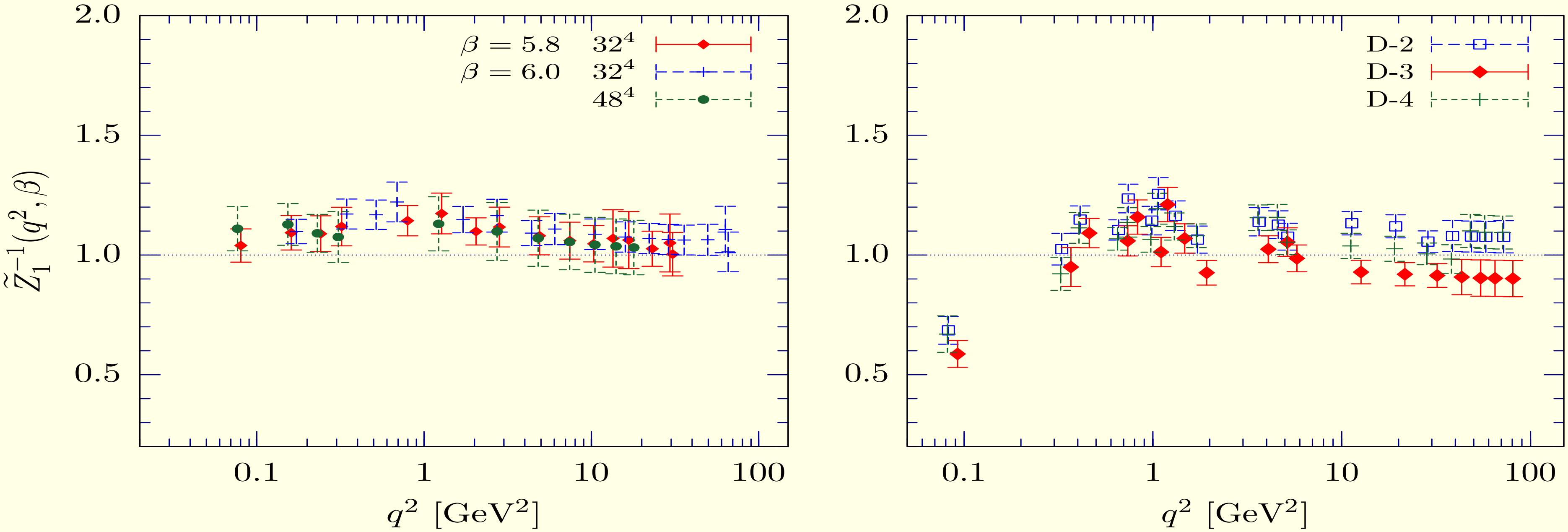
The running coupling from ghost-ghost-gluon vertex

$$\alpha_s(q^2) = \frac{g_0^2}{4\pi} Z(q^2)\,(J(q^2))^2 \quad \text{assuming} \quad \tilde{Z}_1 = 1$$

[perturbation theory: Taylor ('71) / LGT $SU(2)$: Cucchieri et al. ('04)]

The $SU(3)$ vertex renormalization function $\boldsymbol{Z_1}$, gluon momentum $k = 0$.

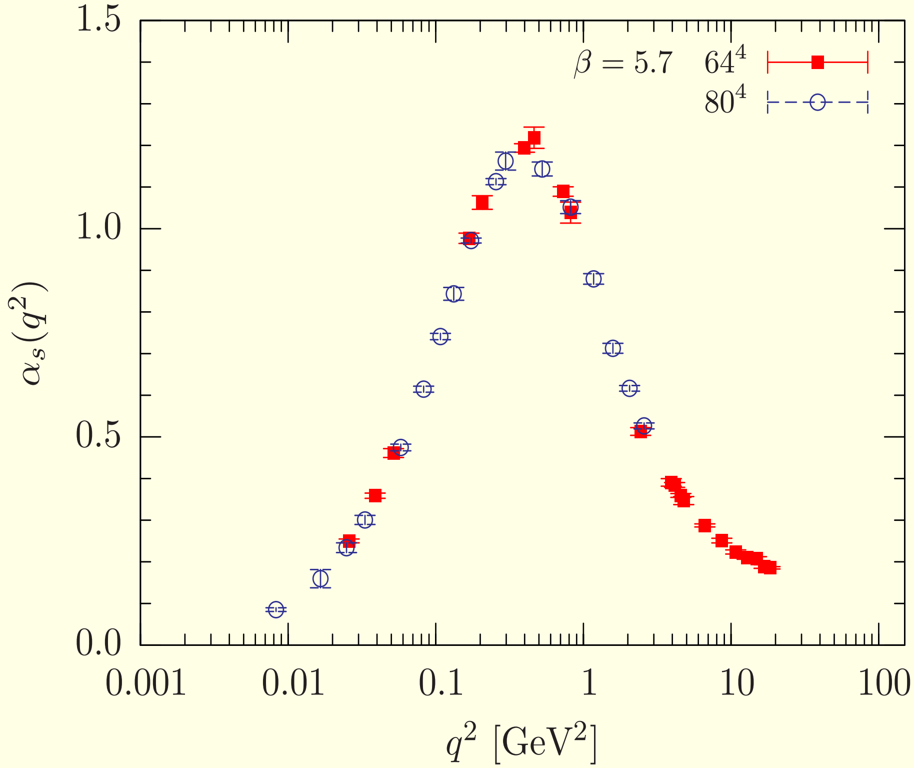
$N_f = 0$ Ilgenfritz et al. '05 $N_f = 2$ Sternbeck thesis '06



Result for the running coupling on large volumes

quenched QCD, first but long run SA copies, coarse lattices

[Bogolubsky, Ilgenfritz, M.-P., Sternbeck, PLB 676 (2009)]



- Running coupling not monotonous, $\alpha_s \to 0$ for $q \to 0$,
  $\Longrightarrow$ "decoupling behaviour".
- Agrees with other lattice studies, in particular for the three-gluon vertex.
- At large q^2 allows to fix $\Lambda_{\overline{MS}}$.

4. Gribov copies, continuum limit, finite-volume effects, multiplicative renormalization as seen in $SU(2)$ LGT

Systematic effects somewhat easier to study in pure $SU(2)$ gauge theory.
[Bakeev, Bogolubsky, Bornyakov, Burgio, Ilgenfritz, Mitrjushkin, M.-P. ('04 - '09)]

Improved gauge fixing $\Longrightarrow$ getting 'close' to the FMR:

- Simulated annealing (SA):

  Find g's randomly with statistical weight:

$$W \;\propto\; \exp\left(\frac{F_U(g)}{T}\right) .$$

Let "temperature" T slowly decrease. Infinitely slow cooling ends at the global extremum. In practice SA clearly wins for large lattice sizes. (Over)relaxation (OR) has to be applied subsequently in order to reach

$$(\partial\mathcal{A})_x = \sum_{\mu=1}^{4}\left(\mathcal{A}_{x+\hat{\mu}/2;\mu} - \mathcal{A}_{x-\hat{\mu}/2;\mu}\right) < \epsilon \quad \text{for all} \;\; x\,.$$

- $\mathbb{Z}(N)$ flips:

  Gauge functional $F_U(g)$ maximized by enlarging the gauge orbit with respect to $\mathbb{Z}(N)$ non-periodic gauge transformations:

$$g(x + L\hat{\nu}) = z_\nu g(x) \,, \quad z_\nu \in \mathbb{Z}(N) \,.$$

  For $SU(N)$ the N^4 different sectors of Polyakov loop averages are combined.

In order to view Gribov copy effects we compare:

i) standard method: first (i.e. random) copy overrelaxation = "fc OR",

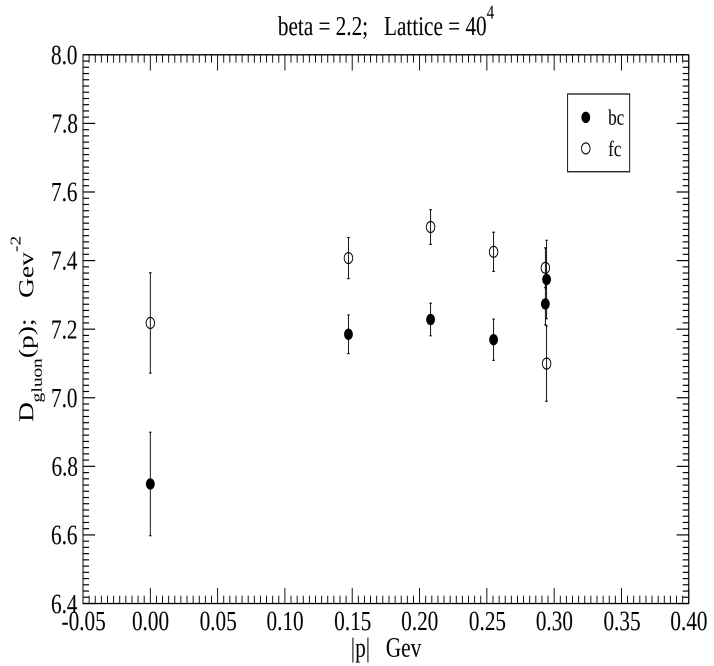
ii) first copy simulated annealing (incl. finalizing overrelaxation) = "fc SA",

iii) best copy $\mathbb{Z}(2)$ flips + simulated annealing (+OR) = "bc FSA",
compare typically 5 copies in each of the 16 Polyakov loop sectors
(= 80 copies).

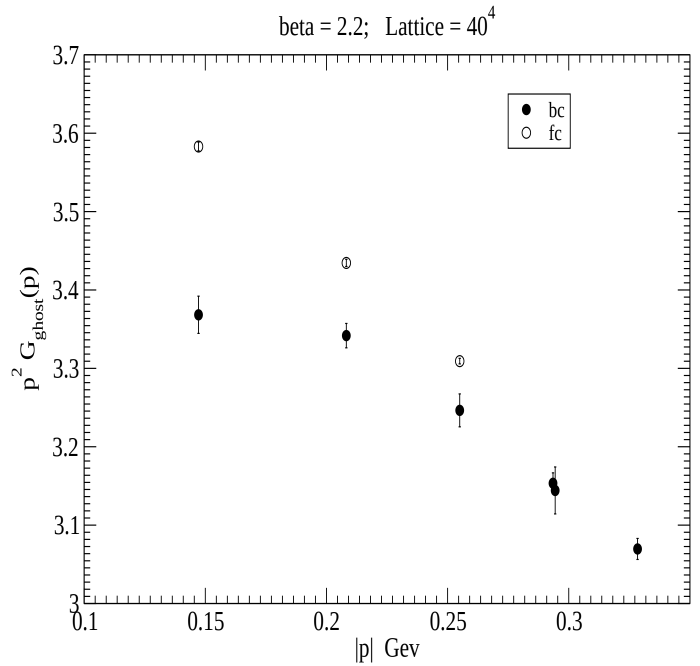
Gluon propagator and ghost dressing fct.: fc SA versus bc FSA

[Bornyakov, Mitrjushkin, M.-P., PRD 79 (2009), arXiv:0812.2761 [hep-lat]]

gluon propagator



ghost dress. fct.



$\Rightarrow$ Gribov copies important for both gluon and ghost!

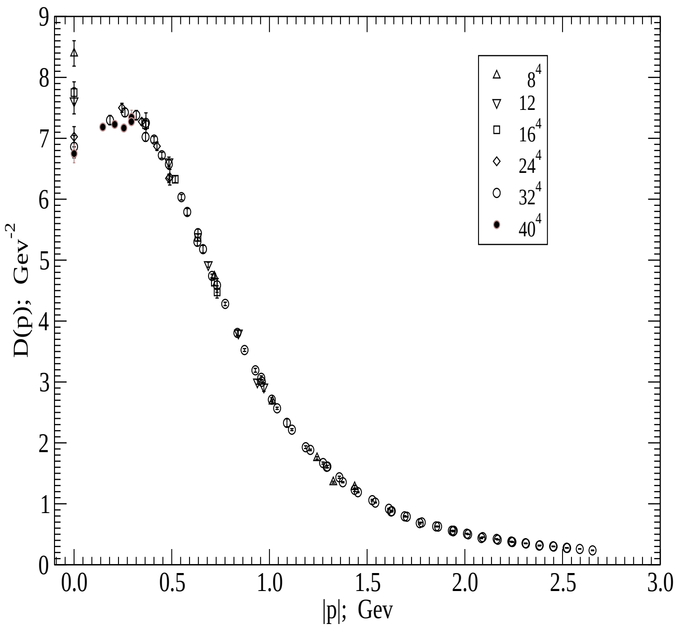
$\Rightarrow$ The closer to the global maximum (FMR), the weaker the 'singularity' of the ghost dressing fct., the lower the IR values of the gluon propagator.

$\Rightarrow$ $D(q^2 \to 0) = 0$? Together with a non-singular ghost dress. fct. this would completely contradict DS and FRG eqs. and (modified) Zwanziger approach.

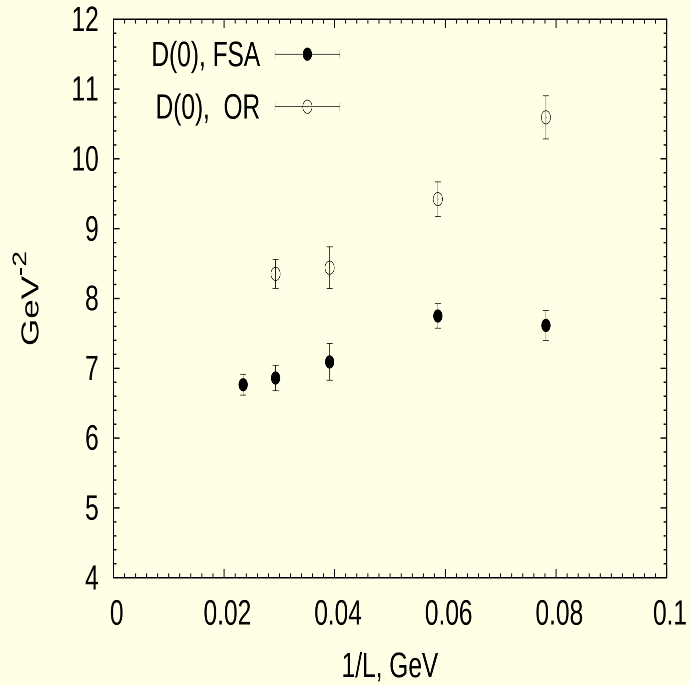
Gluon propagator $\beta = 2.2$: bc FSA versus fc OR

[Bornyakov, Mitrjushkin, M.-P., PRD 79 (2009), arXiv:0812.2761 [hep-lat]]

bc FSA $D(q^2)$ for diff. L^4



$D(0)$ versus $1/L$



⇒ Finite-size effects weaker for FSA, i.e. when approaching the FMR Λ.

⇒ Extrapolation $D(0) \neq 0$ for $V \to \infty$.

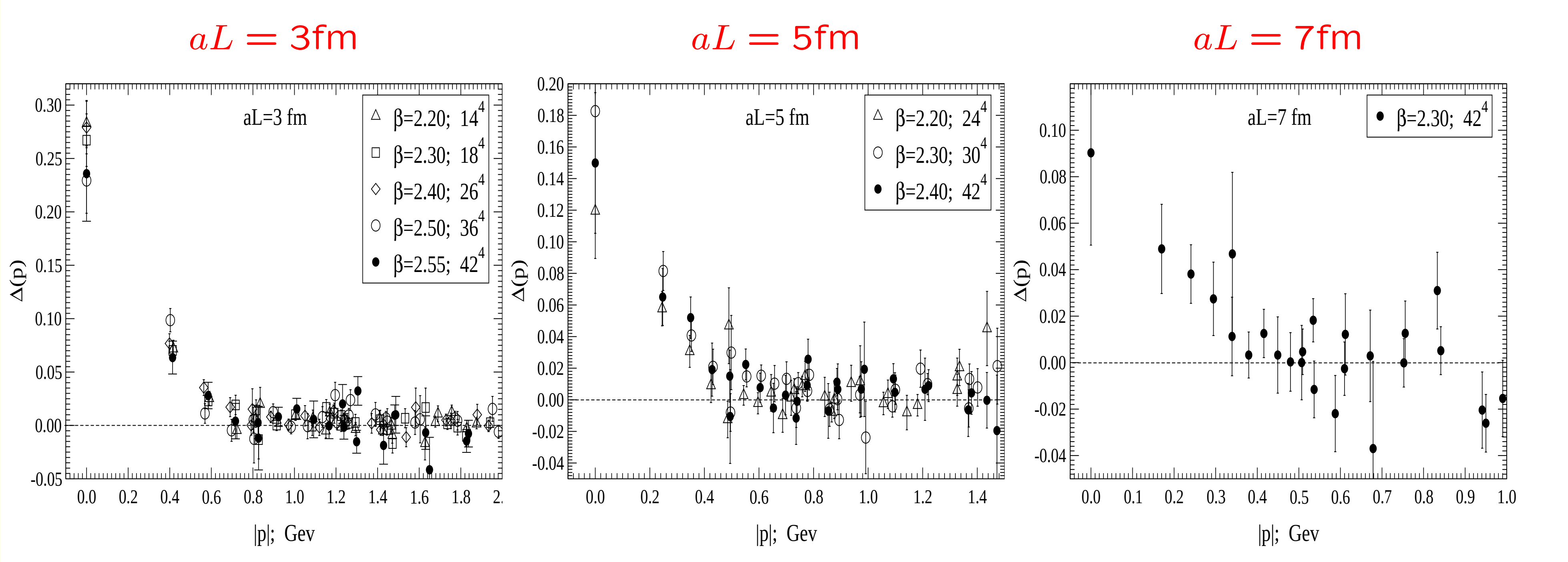
⇒ Again support for "decoupling solution".

But still we have strong coupling, i.e. coarse lattices.

Gribov copy sensitivity for the gluon propagator bc FSA versus fc SA

$$\Delta(p) = \frac{D^{fc}(p) - D^{bc}(p)}{D^{bc}(p)}$$

[Bornyakov, Mitrjushkin, M.-P., arXiv:0912.4475 [hep-lat]].

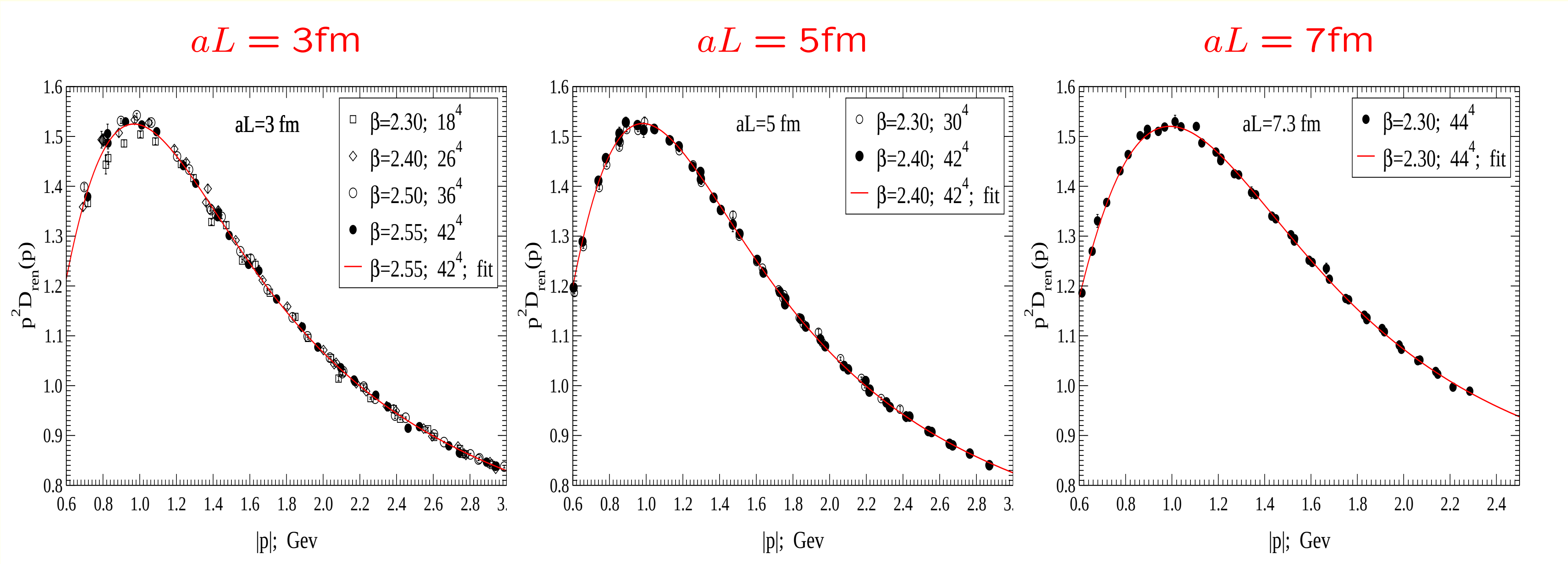


Gribov copy effect: ⇒ important at low momenta,

⇒ almost independent of lattice spacing,

⇒ weakens with increasing physical volume [Zwanziger ('04)].

Finite-volume results for renormalized gluon dressing fct. bc FSA
[Bornyakov, Mitrjushkin, M.-P., arXiv:0912.4475 [hep-lat]].



⇒ For momenta $p > 0.3$MeV, $\beta \geq 2.40$ the renormalized data fall onto each other,

⇒ continuum result reached, good fits available.

⇒ Curves for different linear sizes $3, 5, 7$ fm nicely agree.

5. Conclusion and outlook

- Lattice results support "decoupling solution" as long as we assume approach

$$F_U(g) \rightarrow \text{Global Max.}$$

- Gribov effects turn out to be important. For pure LGT simulated annealing + $\mathbb{Z}(N)$ flips ("bc FSA") provides a solution with weak finite-size effects.

- Continuum limit can be reached within the non-perturbatively and phenomenologically important range around 1GeV.

- Analogous results, when available for full QCD will allow to tune DS and FRG truncations and provide immediate input into Bethe-Salpeter or Faddeev Eqs.

- Debate "scaling" versus "decoupling" solution continues.
  On the lattice it could mean to give up the condition $F_U(g) \rightarrow$ Global Max.
  [Maas et al. ('09)]

- Continuum alternative ?
  $\Longrightarrow$ A.A. Slavnov – Y.-M. theory without Gribov ambiguity

Thank you for your attention.

Best wishes to

Andrei Alekseevich.