Correlation functions of the XXZ spin chain. Form factor approach

N. Kitanine, K. K. Kozlowski, J. M. Maillet, N. A. Slavnov, V. Terras

[1] *Algebraic Bethe ansatz approach to the asymptotic behavior of correlation functions*, J. Stat. Mech. (2009) P04003.

[2] *On the thermodynamic limit of form factors in the massless XXZ Heisenberg chain*, J. Math. Phys. **50**, Iss. 9, (2009) 095209.

Heisenberg spin chains

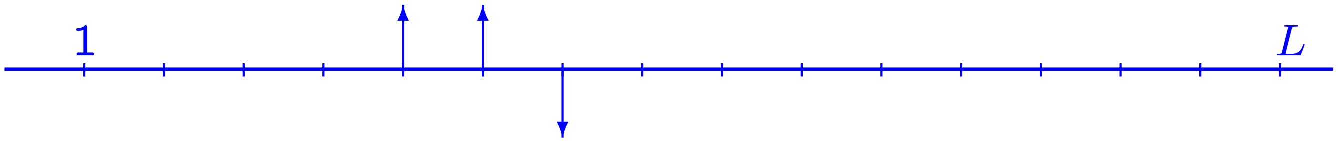
$$H = \sum_{m=1}^{L} \left(J_x S_m^x S_{m+1}^x + J_y S_m^y S_{m+1}^y + J_z S_m^z S_{m+1}^z \right)$$

$J_x \neq J_y \neq J_z$ — XYZ chain

$J_x = J_y \neq J_z$ — XXZ chain

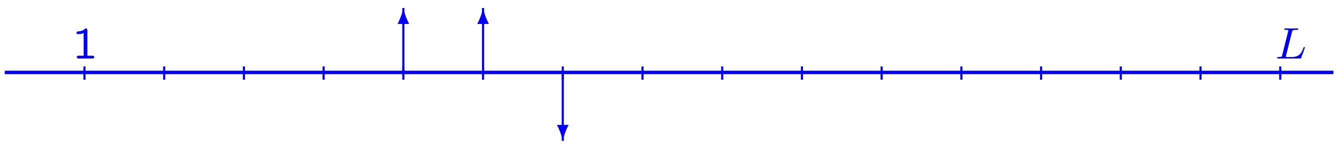
$J_x = J_y = J_z$ — XXX chain

XXZ spin-$1/2$ chain



$$H = \sum_{m=1}^{L} \left(\sigma_m^x \sigma_{m+1}^x + \sigma_m^y \sigma_{m+1}^y + \Delta \sigma_m^z \sigma_{m+1}^z \right)$$

XXZ spin-$1/2$ chain

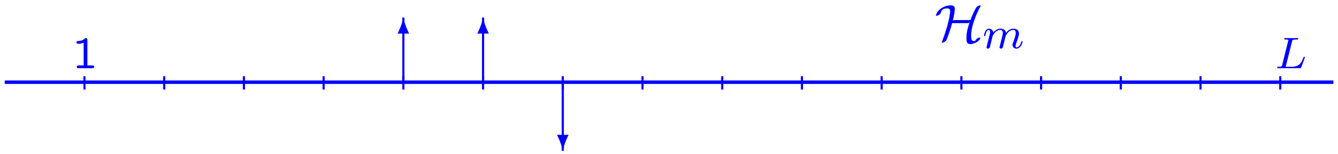


$$H = \sum_{m=1}^{L} \left(\sigma_m^x \sigma_{m+1}^x + \sigma_m^y \sigma_{m+1}^y + \Delta \sigma_m^z \sigma_{m+1}^z \right)$$

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma^{\pm} = \frac{1}{2}(\sigma^x \pm i\sigma^y)$$

XXZ spin-1/2 chain



$$H = \sum_{m=1}^{L} \left(\sigma_m^x \sigma_{m+1}^x + \sigma_m^y \sigma_{m+1}^y + \Delta \sigma_m^z \sigma_{m+1}^z \right)$$

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\sigma^{\pm} = \frac{1}{2}(\sigma^x \pm i\sigma^y)$$

$$\mathcal{H} = \bigotimes_{m=1}^{L} \mathcal{H}_m \qquad \sigma_m^{\alpha} = I \otimes \cdots \otimes \sigma^{\alpha} \otimes \cdots \otimes I$$

Eigenvectors $H|\psi\rangle = E|\psi\rangle$

Matrix elements $\langle\psi'|\mathcal{O}(\sigma_m^\alpha)|\psi\rangle$

Eigenvectors $H|\psi\rangle = E|\psi\rangle$

Matrix elements $\langle\psi'|\mathcal{O}(\sigma_m^\alpha)|\psi\rangle$

Two-point functions

$$\langle\psi|\sigma_1^z\sigma_{m+1}^z|\psi\rangle \qquad \langle\psi|\sigma_1^-\sigma_{m+1}^+|\psi\rangle \qquad \langle\psi|\sigma_1^+\sigma_{m+1}^-|\psi\rangle$$

Ground state two-point functions in the thermodynamic limit $(L \to \infty)$

$$\langle \sigma_1^\alpha \sigma_{m+1}^\beta \rangle = \lim_{L \to \infty} \langle \psi_g | \sigma_1^\alpha \sigma_{m+1}^\beta | \psi_g \rangle$$

$$H|\psi_g\rangle = E_{min}|\psi_g\rangle$$

Ground state two-point functions in the thermodynamic limit ($L \to \infty$)

$$\langle \sigma_1^\alpha \sigma_{m+1}^\beta \rangle = \lim_{L\to\infty} \langle \psi_g | \sigma_1^\alpha \sigma_{m+1}^\beta | \psi_g \rangle$$

$$H|\psi_g\rangle = E_{min}|\psi_g\rangle$$

$$\langle \psi_g | \sigma_1^\alpha \sigma_{m+1}^\beta | \psi_g \rangle = f^{(\alpha,\beta)}(m, \Delta, L) = f_0^{(\alpha,\beta)}(m, \Delta) + \frac{1}{L} f_1^{(\alpha,\beta)}(m, \Delta) + \ldots$$

Ground state two-point functions in the thermodynamic limit ($L \to \infty$)

$$\langle \sigma_1^\alpha \sigma_{m+1}^\beta \rangle = \lim_{L\to\infty} \langle \psi_g | \sigma_1^\alpha \sigma_{m+1}^\beta | \psi_g \rangle$$

$$H|\psi_g\rangle = E_{min}|\psi_g\rangle$$

$$\langle \psi_g | \sigma_1^\alpha \sigma_{m+1}^\beta | \psi_g \rangle = f^{(\alpha,\beta)}(m,\Delta,L) = f_0^{(\alpha,\beta)}(m,\Delta) + \frac{1}{L} f_1^{(\alpha,\beta)}(m,\Delta) + \dots$$

$$\langle \sigma_1^\alpha \sigma_{m+1}^\beta \rangle = f_0^{(\alpha,\beta)}(m,\Delta) = \lim_{L\to\infty} \langle \psi_g | \sigma_1^\alpha \sigma_{m+1}^\beta | \psi_g \rangle$$

Long distance asymptotic behavior $m \to \infty$

$$\langle \sigma_1^\alpha \sigma_{m+1}^\beta \rangle = f_0^{(\alpha,\beta)}(m, \Delta), \qquad m \to \infty$$

Massive phase $(\Delta > 1)$**:** $\langle \sigma_1^\alpha \sigma_{m+1}^\beta \rangle \sim C\, e^{-m/r}, \qquad m \to \infty$

Massless phase $(|\Delta| < 1)$**:** $\langle \sigma_1^\alpha \sigma_{m+1}^\beta \rangle \sim C\, m^{-\theta}, \qquad m \to \infty$

Two-point correlation functions in the thermodynamic limit

$$\langle \sigma_1^{\alpha} \sigma_{m+1}^{\beta} \rangle = \sum_{n=0}^{\infty} \int G_n^{(\alpha,\beta)}(m|\lambda_1, \ldots, \lambda_n) d\lambda_1 \cdots d\lambda_n$$

$$m \to \infty$$

Two-point correlation functions in the thermodynamic limit

$$\langle \sigma_1^\alpha \sigma_{m+1}^\beta \rangle = \sum_{n=0}^{\infty} \int G_n^{(\alpha,\beta)}(m|\lambda_1, \ldots, \lambda_n) d\lambda_1 \cdots d\lambda_n$$

$$m \rightarrow \infty$$

Form factors at L finite

$$\langle \psi' | \sigma_m^\alpha | \psi \rangle = e^{im\mathcal{P}} \det M^{(\alpha)}$$

$$L \rightarrow \infty$$

Bethe Ansatz (1931)

$$H|\psi\rangle = E|\psi\rangle$$

$$\mathcal{H} = \bigoplus_{N=0}^{L} \mathcal{H}^{(N)} \qquad\qquad H \cdot \mathcal{H}^{(N)} \to \mathcal{H}^{(N)}$$

N is a number of spins down in $|\psi\rangle$

magnetization : $1 - \dfrac{2N}{L}$

Bethe Ansatz (1931)

$$H|\psi\rangle = E|\psi\rangle$$

$$|\psi\rangle = |\psi(\lambda_1, \ldots, \lambda_N)\rangle, \qquad N = 0, 1, \ldots, L$$

Bethe Ansatz (1931)

$$H|\psi\rangle = E|\psi\rangle$$

$$|\psi\rangle = |\psi(\lambda_1, \ldots, \lambda_N)\rangle, \qquad N = 0, 1, \ldots, L$$

$$e^{iLp_0(\lambda_j)} \prod_{k=1}^{N} \frac{\sinh(\lambda_j - \lambda_k + i\zeta)}{\sinh(\lambda_j - \lambda_k - i\zeta)} = -1, \qquad j = 1, \ldots, N$$

$$p_0(\lambda) = i \log \left(\frac{\sinh(\frac{i\zeta}{2} + \lambda)}{\sinh(\frac{i\zeta}{2} - \lambda)} \right), \qquad \Delta = \cos\zeta$$

Bethe Ansatz (1931)

$$H|\psi\rangle = E|\psi\rangle$$

$$|\psi\rangle = |\psi(\lambda_1, \ldots, \lambda_N)\rangle, \qquad N = 0, 1, \ldots, L$$

$$E = -\sum_{k=1}^{N} \frac{\sin^2 \zeta}{\sinh(\lambda_k + \frac{i\zeta}{2}) \sinh(\lambda_k - \frac{i\zeta}{2})}, \qquad \Delta = \cos \zeta$$

Bethe Ansatz (1931)

$$H|\psi\rangle = E|\psi\rangle$$

$$|\psi\rangle = |\psi(\lambda_1, \ldots, \lambda_N)\rangle, \qquad N = 0, 1, \ldots, L$$

$$e^{iLp_0(\lambda_j)} \prod_{k=1}^{N} \frac{\sinh(\lambda_j - \lambda_k + i\zeta)}{\sinh(\lambda_j - \lambda_k - i\zeta)} = -1, \qquad j = 1, \ldots, N$$

$$p_0(\lambda) = i \log \left(\frac{\sinh(\frac{i\zeta}{2} + \lambda)}{\sinh(\frac{i\zeta}{2} - \lambda)} \right), \qquad \Delta = \cos\zeta$$

Free fermions

$$e^{iLp_0(\lambda_j)} \prod_{k=1}^{N} \frac{\sinh(\lambda_j - \lambda_k + i\zeta)}{\sinh(\lambda_j - \lambda_k - i\zeta)} = -1, \qquad j = 1, \ldots, N$$

Free fermions

$$e^{iLp_0(\lambda_j)} \prod_{k=1}^{N} \frac{\sinh(\lambda_j - \lambda_k + i\zeta)}{\sinh(\lambda_j - \lambda_k - i\zeta)} = -1, \qquad j = 1, \ldots, N$$

$$\Delta = 0 \qquad \Longleftrightarrow \qquad \zeta = \frac{\pi}{2}$$

Free fermions

$$e^{iLp_0(\lambda_j)} \prod_{k=1}^{N} \frac{\sinh(\lambda_j - \lambda_k + i\zeta)}{\sinh(\lambda_j - \lambda_k - i\zeta)} = -1, \qquad j = 1, \ldots, N$$

$$\Delta = 0 \qquad \Longleftrightarrow \qquad \zeta = \frac{\pi}{2}$$

$$e^{iLp_0(\lambda_j)} = (-1)^{N-1}, \qquad p_0(\lambda_j) = \frac{2\pi}{L}\ell_j, \qquad -\frac{L}{2} < \ell_j \leq \frac{L}{2}$$

The eigenstates $|\psi(\lambda_1, \ldots, \lambda_N)\rangle$ are parameterized by sets of pair-wise distinct (half)integers ℓ_j

Logarithmic form of Bethe equations

$$L p_0(\lambda_j) + i \sum_{k=1}^{N} \log\left(\frac{\sinh(i\zeta - \lambda_j + \lambda_k)}{\sinh(i\zeta + \lambda_j - \lambda_k)}\right) = 2\pi \ell_j$$

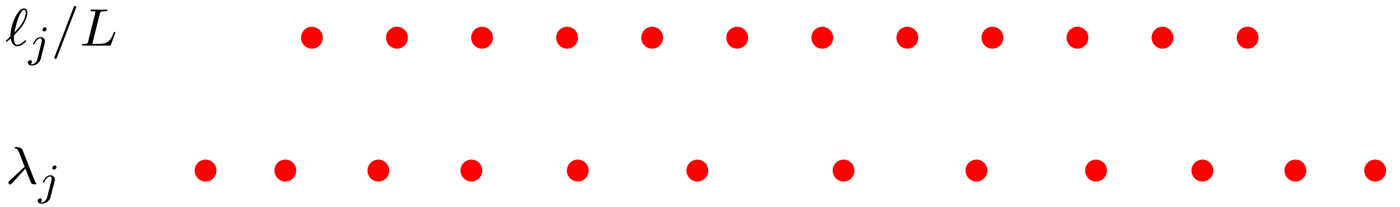
ℓ_j are integers if N is odd

ℓ_j are half-integers if N is even

Logarithmic form of Bethe equations

$$L p_0(\lambda_j) + i \sum_{k=1}^{N} \log\left(\frac{\sinh(i\zeta - \lambda_j + \lambda_k)}{\sinh(i\zeta + \lambda_j - \lambda_k)}\right) = 2\pi \ell_j$$

Ground state: $\ell_j = j - \dfrac{N+1}{2}$

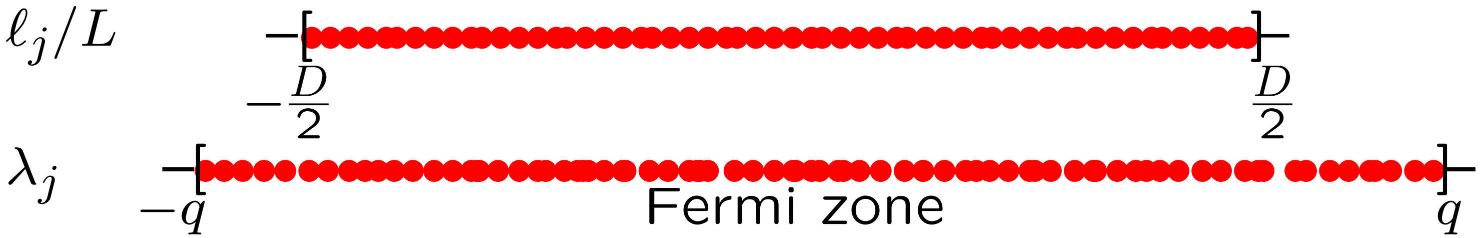


Logarithmic form of Bethe equations

$$Lp_0(\lambda_j) + i \sum_{k=1}^{N} \log \left(\frac{\sinh(i\zeta - \lambda_j + \lambda_k)}{\sinh(i\zeta + \lambda_j - \lambda_k)} \right) = 2\pi \ell_j$$

Ground state: $\ell_j = j - \dfrac{N+1}{2}$

Thermodynamic limit : $L, N \to \infty, \qquad N/L = D$



$\lambda_1 \to -q, \qquad \lambda_N \to q$: Fermi boundaries

Two-point correlation functions in the thermodynamic limit

$$\langle \sigma_1^{\alpha} \sigma_{m+1}^{\beta} \rangle = \sum_{n=0}^{\infty} \int_{-q}^{q} G_n^{(\alpha,\beta)}(m|\lambda_1, \ldots, \lambda_n) d\lambda_1 \cdots d\lambda_n$$

$$m \to \infty$$

Form factors at L finite

$$\langle \psi' | \sigma_m^{\alpha} | \psi \rangle = e^{im\mathcal{P}} \det M^{(\alpha)}$$

$$L \to \infty$$

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = C_0 m^{-\theta_0} \left(1 + O(m^{-1})\right)$$

$$+C_1 e^{imp} m^{-\theta_1} \left(1 + O(m^{-1})\right) + C_{-1} e^{-imp} m^{-\theta_{-1}} \left(1 + O(m^{-1})\right)$$

$$+C_2 e^{2imp} m^{-\theta_2} \left(1 + O(m^{-1})\right) + C_{-2} e^{-2imp} m^{-\theta_{-2}} \left(1 + O(m^{-1})\right) + \ldots$$

$$0 < \theta_0 < \theta_{\pm 1} < \theta_{\pm 2} < \ldots$$

The constants p, θ_k, C_k in the asymptotic behavior of $\langle\sigma_1^-\sigma_{m+1}^+\rangle$ can be found from the analysis of the of the thermodynamic limit of the form factors:

$$\langle\psi|\sigma_1^-|\psi'\rangle \qquad \langle\psi'|\sigma_{m+1}^+|\psi\rangle$$

Moreover the constants p, θ_k can be found from the analysis of the spectral data (Bethe equations).

Form factors of $\sigma_m^{\pm}$

$$\langle\psi(\lambda_1,\ldots,\lambda_N)|\sigma_m^-|\psi(\lambda'_1,\ldots,\lambda'_{N+1})\rangle = \mathcal{F}^{(-)}_{\ell,\ell'}(m)$$

$$\langle\psi(\lambda'_1,\ldots,\lambda'_{N+1})|\sigma_m^+|\psi(\lambda_1,\ldots,\lambda_N)\rangle = \mathcal{F}^{(+)}_{\ell',\ell}(m)$$

Form factors of $\sigma_m^{\pm}$

$$\langle\psi(\lambda_1,\ldots,\lambda_N)|\sigma_m^-|\psi(\lambda'_1,\ldots,\lambda'_{N+1})\rangle = \mathcal{F}^{(-)}_{\ell,\ell'}(m)$$

$$\langle\psi(\lambda'_1,\ldots,\lambda'_{N+1})|\sigma_m^+|\psi(\lambda_1,\ldots,\lambda_N)\rangle = \mathcal{F}^{(+)}_{\ell',\ell}(m)$$

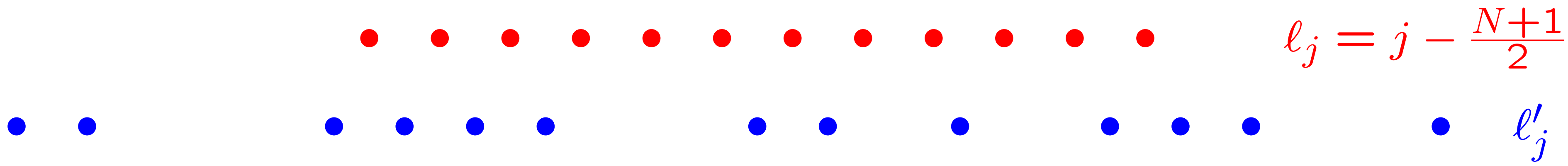
$$Lp_0(\lambda_j) + i\sum_{k=1}^{N}\log\left(\frac{\sinh(i\zeta-\lambda_j+\lambda_k)}{\sinh(i\zeta+\lambda_j-\lambda_k)}\right) = 2\pi\ell_j$$

$$Lp_0(\lambda'_j) + i\sum_{k=1}^{N+1}\log\left(\frac{\sinh(i\zeta-\lambda'_j+\lambda'_k)}{\sinh(i\zeta+\lambda'_j-\lambda'_k)}\right) = 2\pi\ell'_j$$

Form factors of $\sigma_m^{\pm}$

$$\langle \psi(\lambda_1, \ldots, \lambda_N) | \sigma_m^- | \psi(\lambda'_1, \ldots, \lambda'_{N+1}) \rangle = \mathcal{F}^{(-)}_{\ell,\ell'}(m)$$

$$\langle \psi(\lambda'_1, \ldots, \lambda'_{N+1}) | \sigma_m^+ | \psi(\lambda_1, \ldots, \lambda_N) \rangle = \mathcal{F}^{(+)}_{\ell',\ell}(m)$$

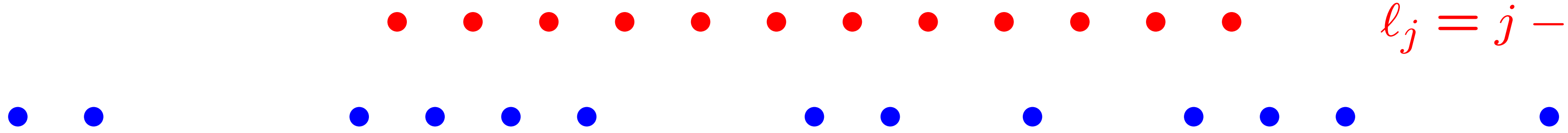


Form factors of $\sigma_m^{\pm}$

$$\langle \psi(\lambda_1, \ldots, \lambda_N) | \sigma_m^- | \psi(\lambda'_1, \ldots, \lambda'_{N+1}) \rangle = \mathcal{F}^{(-)}_{\ell,\ell'}(m)$$

$$\langle \psi(\lambda'_1, \ldots, \lambda'_{N+1}) | \sigma_m^+ | \psi(\lambda_1, \ldots, \lambda_N) \rangle = \mathcal{F}^{(+)}_{\ell',\ell}(m)$$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = \sum_{\{\ell'\}} \mathcal{F}^{(-)}_{\ell,\ell'}(1) \mathcal{F}^{(+)}_{\ell',\ell}(m+1)$$



$\ell_j = j - \frac{N+1}{2}$

ℓ'_j

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = C_0 m^{-\theta_0} \left(1 + O(m^{-1})\right)$$

$$+ C_1 e^{imp} m^{-\theta_1} \left(1 + O(m^{-1})\right) + C_{-1} e^{-imp} m^{-\theta_{-1}} \left(1 + O(m^{-1})\right)$$

$$+ C_2 e^{2imp} m^{-\theta_2} \left(1 + O(m^{-1})\right) + C_{-2} e^{-2imp} m^{-\theta_{-2}} \left(1 + O(m^{-1})\right) + \dots$$

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = \boxed{C_0 m^{-\theta_0} \left(1 + O(m^{-1})\right)}$$

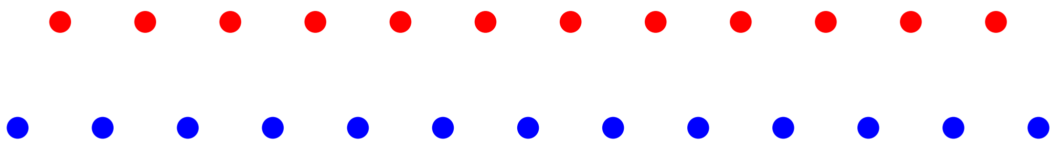
$$+C_1 e^{imp} m^{-\theta_1} \left(1 + O(m^{-1})\right) + C_{-1} e^{-imp} m^{-\theta_{-1}} \left(1 + O(m^{-1})\right)$$

$$+C_2 e^{2imp} m^{-\theta_2} \left(1 + O(m^{-1})\right) + C_{-2} e^{-2imp} m^{-\theta_{-2}} \left(1 + O(m^{-1})\right) + \ldots$$

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = \boxed{C_0 m^{-\theta_0} \left(1 + O(m^{-1})\right)} + \dots$$

$$\mathcal{F}^{(-)}_{\ell,\ell'}(1)\mathcal{F}^{(+)}_{\ell',\ell}(m+1) \to C_0 \left(\frac{L}{2\pi}\right)^{-\theta_0} \left(1 + O(L^{-1})\right), \qquad L, N \to \infty$$



$$\ell_j = j - \frac{N+1}{2}$$

$$\ell'_j = j - \frac{N+2}{2}$$

$$\mathcal{E}_{ex} = 0 \qquad \mathcal{P}_{ex} = 0$$

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = C_0 m^{-\theta_0} \left(1 + O(m^{-1})\right)$$

$$+C_1 e^{imp} m^{-\theta_1} \left(1 + O(m^{-1})\right) + C_{-1} e^{-imp} m^{-\theta_{-1}} \left(1 + O(m^{-1})\right)$$

$$+C_2 e^{2imp} m^{-\theta_2} \left(1 + O(m^{-1})\right) + C_{-2} e^{-2imp} m^{-\theta_{-2}} \left(1 + O(m^{-1})\right) + \ldots$$

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = C_0 m^{-\theta_0} \left(1 + O(m^{-1})\right)$$

$$\boxed{+C_1 e^{imp} m^{-\theta_1} \left(1 + O(m^{-1})\right)} + C_{-1} e^{-imp} m^{-\theta_{-1}} \left(1 + O(m^{-1})\right)$$

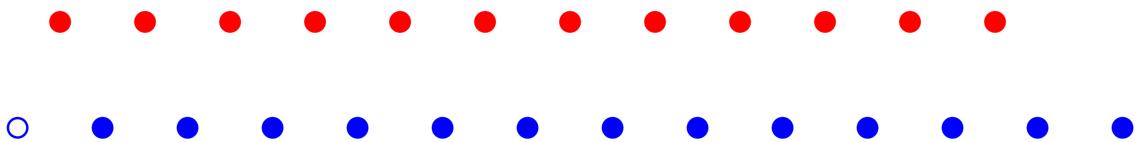
$$+C_2 e^{2imp} m^{-\theta_2} \left(1 + O(m^{-1})\right) + C_{-2} e^{-2imp} m^{-\theta_{-2}} \left(1 + O(m^{-1})\right) + \ldots$$

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = \qquad \dots$$

$$\boxed{+C_1 e^{imp} m^{-\theta_1} \left(1 + O(m^{-1})\right)} + \dots$$

$$\mathcal{F}^{(-)}_{\ell,\ell'}(1)\mathcal{F}^{(+)}_{\ell',\ell}(m+1) \to C_1 e^{imp} \left(\frac{L}{2\pi}\right)^{-\theta_1} \left(1 + O(L^{-1})\right), \qquad L, N \to \infty$$



$$\ell_j = j - \tfrac{N+1}{2}$$

$$\ell'_j = j + 1 - \tfrac{N+2}{2}$$

$$\mathcal{E}_{ex} = 0 \qquad \mathcal{P}_{ex} = p$$

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = C_0 m^{-\theta_0} \left(1 + O(m^{-1})\right)$$

$$+C_1 e^{imp} m^{-\theta_1} \left(1 + O(m^{-1})\right) \boxed{+ C_{-1} e^{-imp} m^{-\theta_{-1}} \left(1 + O(m^{-1})\right)}$$

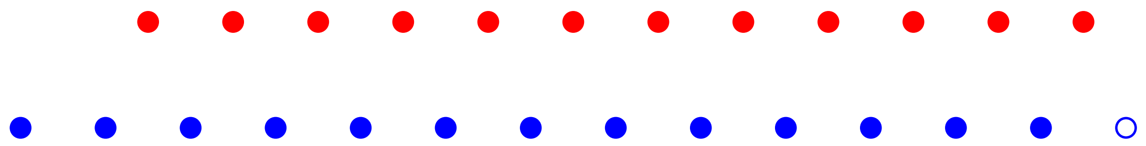
$$+C_2 e^{2imp} m^{-\theta_2} \left(1 + O(m^{-1})\right) + C_{-2} e^{-2imp} m^{-\theta_{-2}} \left(1 + O(m^{-1})\right) + \ldots$$

Asymptotic behavior $m \to \infty$

$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = \qquad \ldots$$

$$+ \qquad \ldots \qquad \boxed{+ C_{-1} e^{-imp} m^{-\theta_{-1}} \left(1 + O(m^{-1})\right)}$$

$$\mathcal{F}^{(-)}_{\ell,\ell'}(1) \mathcal{F}^{(+)}_{\ell',\ell}(m+1) \to C_{-1} e^{-imp} \left(\frac{L}{2\pi}\right)^{-\theta_{-1}} \left(1 + O(L^{-1})\right), \qquad L, N \to \infty$$



$$\ell_j = j - \frac{N+1}{2}$$

$$\ell'_j = j - 1 - \frac{N+2}{2}$$

$$\mathcal{E}_{ex} = 0 \qquad \mathcal{P}_{ex} = -p$$

Asymptotic behavior $m \to \infty$

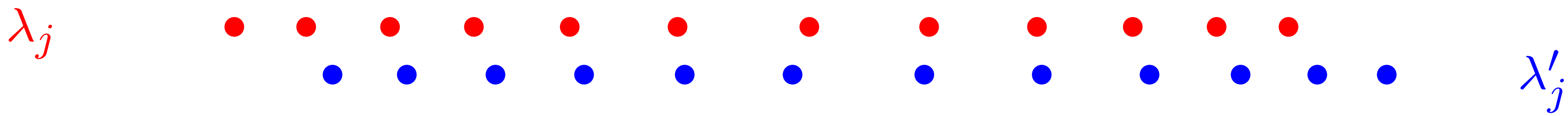
$$\langle \sigma_1^- \sigma_{m+1}^+ \rangle = C_0 m^{-\theta_0} \left(1 + O(m^{-1})\right)$$

$$+ C_1 e^{imp} m^{-\theta_1} \left(1 + O(m^{-1})\right) + C_{-1} e^{-imp} m^{-\theta_{-1}} \left(1 + O(m^{-1})\right)$$

$$+ C_2 e^{2imp} m^{-\theta_2} \left(1 + O(m^{-1})\right) + C_{-2} e^{-2imp} m^{-\theta_{-2}} \left(1 + O(m^{-1})\right) + \ldots$$

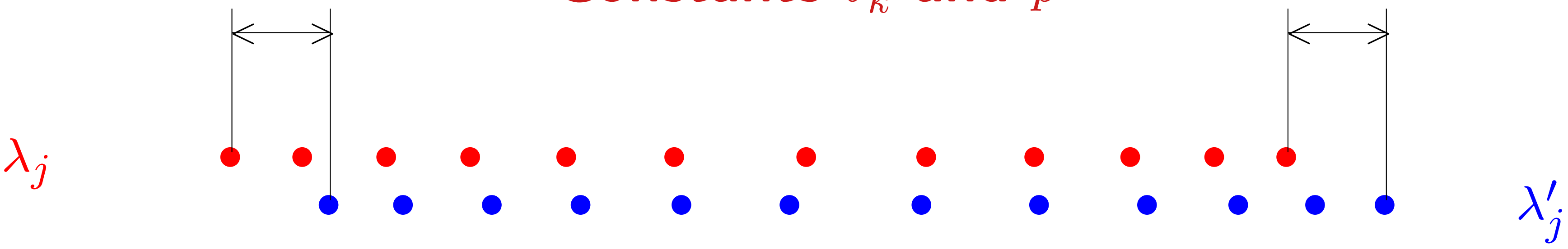
For the calculation of the leading terms in the long distance asymptotic expansion of the two-point function $\langle \sigma_1^- \sigma_{m+1}^+ \rangle$ it is enough to compute the thermodynamic limit of the form factors of $\sigma^\pm$ between the ground state and the states with k particles and k holes on the Fermi boundaries.

Constants θ_k and p



$$\lambda_j = \lambda_j(\{\ell\}) \qquad \lambda'_j = \lambda'_j(\{\ell + k - 1/2\})$$

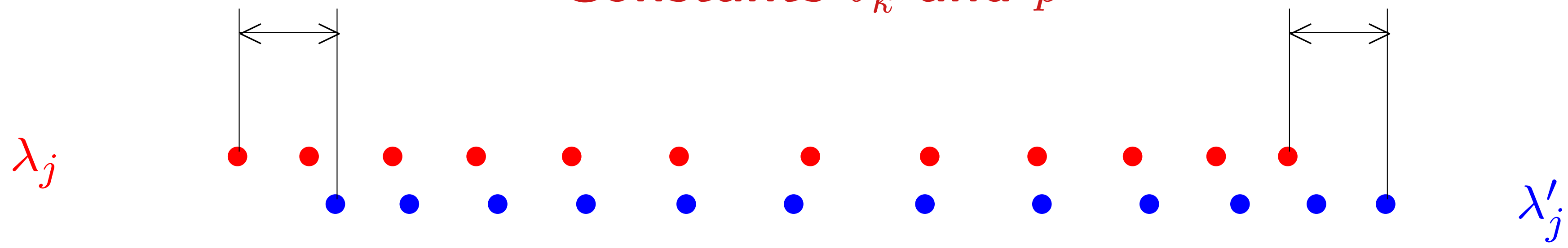
Constants θ_k and p



$$\lambda_j = \lambda_j(\{\ell\}) \qquad \lambda'_j = \lambda'_j(\{\ell + k - 1/2\})$$

$$F(\lambda_j) = \frac{\lambda'_j - \lambda_j}{\lambda_{j+1} - \lambda_j} \qquad \theta_k = F^2(q) + F^2(-q)$$

Constants θ_k and p

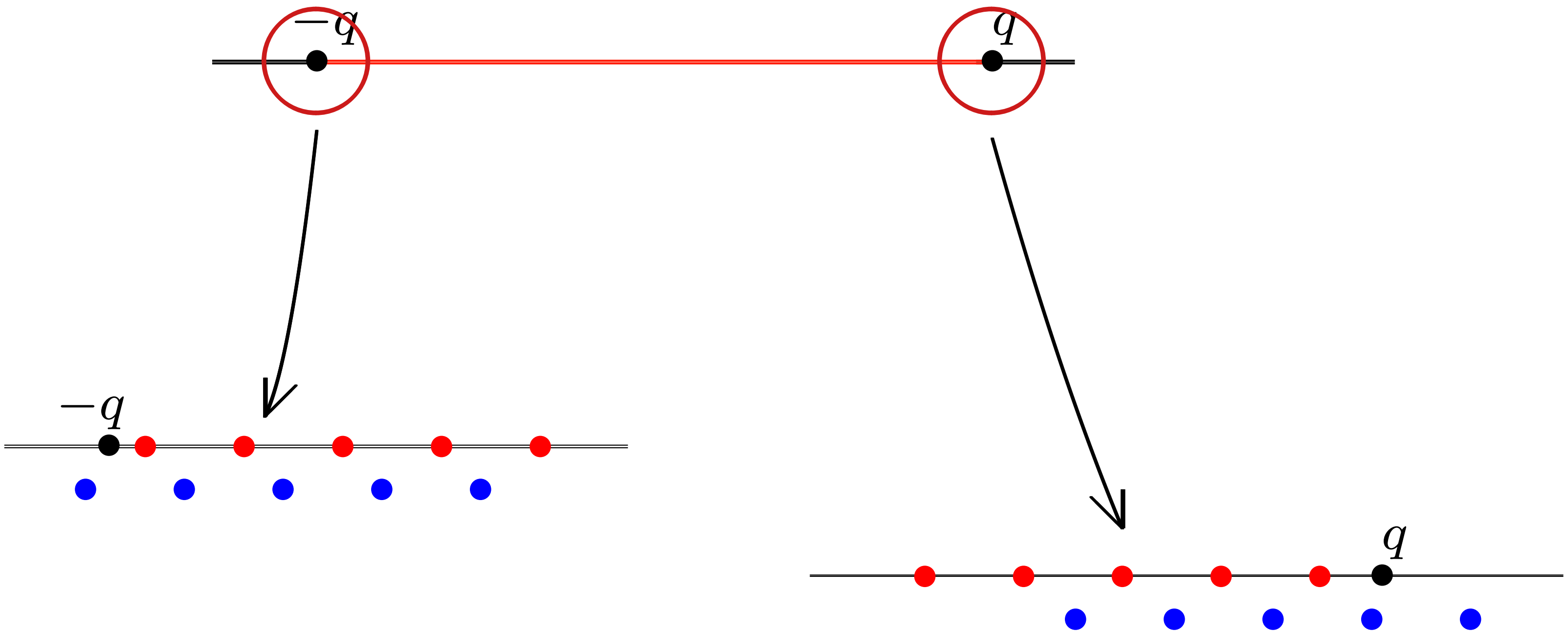


$$\lambda_j = \lambda_j(\{\ell\}) \qquad \lambda'_j = \lambda'_j(\{\ell + k - 1/2\})$$

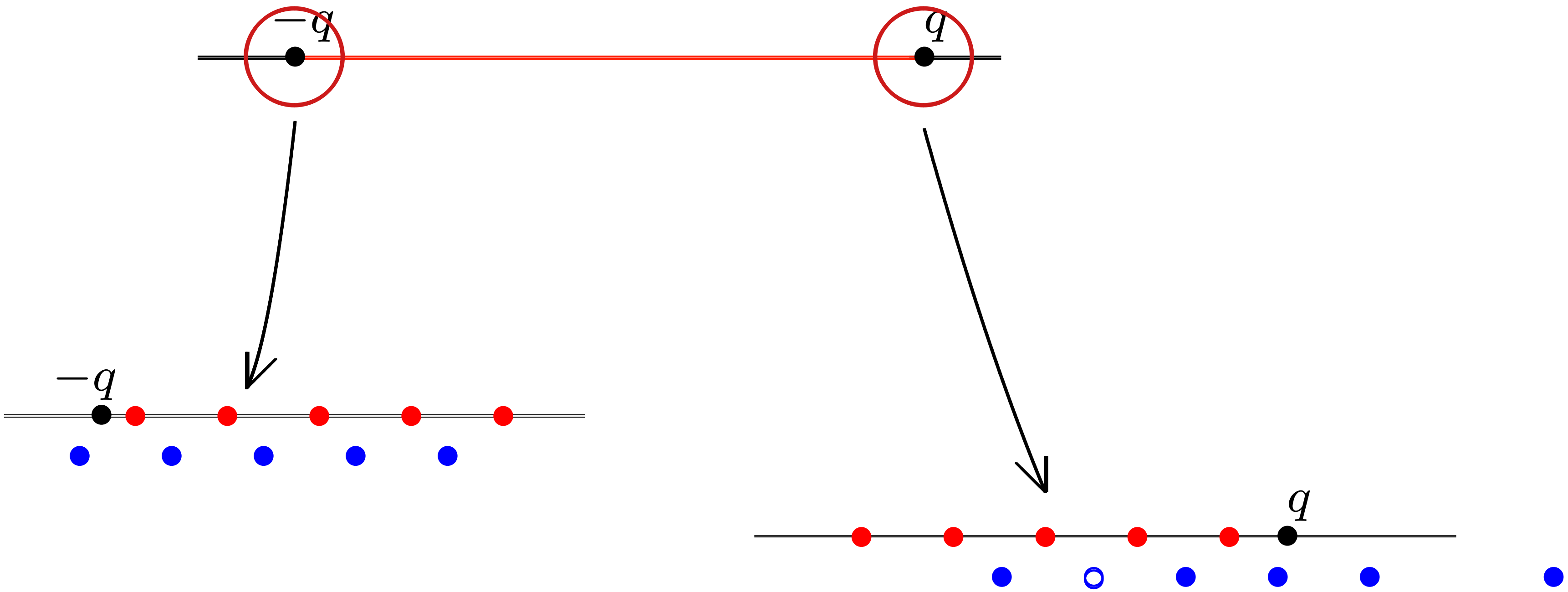
$$F(\lambda_j) = \frac{\lambda'_j - \lambda_j}{\lambda_{j+1} - \lambda_j} \qquad \theta_k = F^2(q) + F^2(-q)$$

$$kp = \mathcal{P}_{ex} = \sum_{j=1}^{N} \left(p_0(\lambda'_j) - p_0(\lambda_j) \right)$$

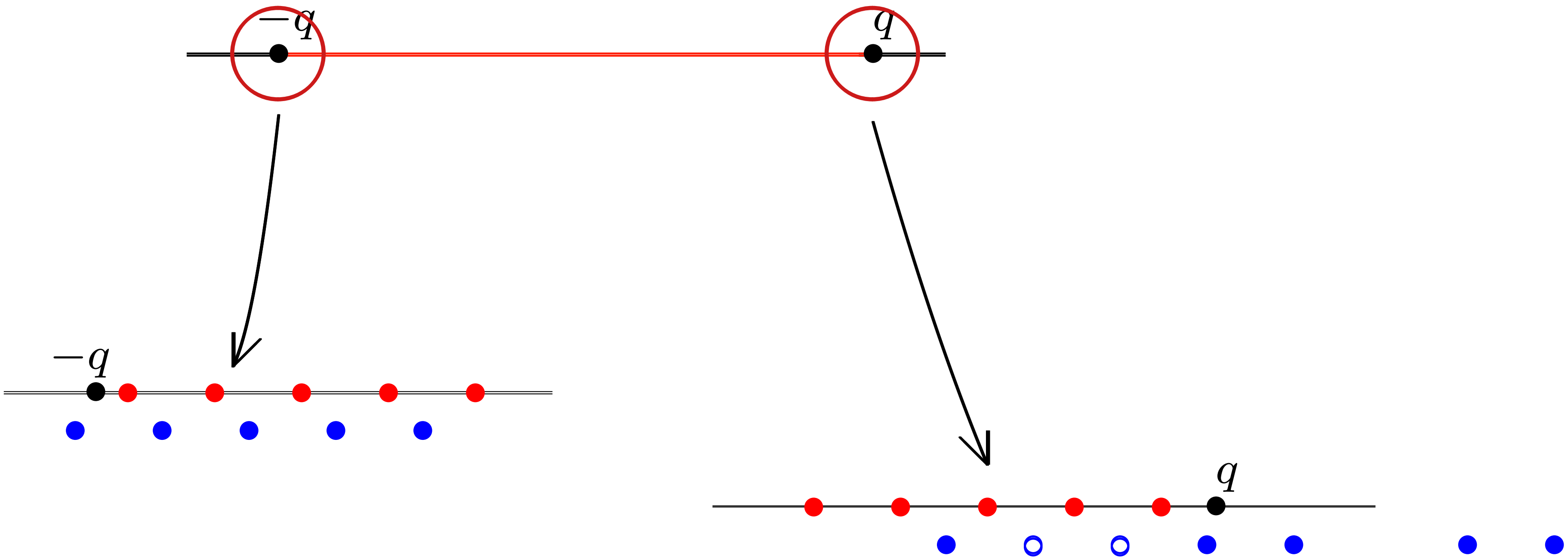
The same relation between two-point functions and form factors exists for other models solvable by Bethe Ansatz



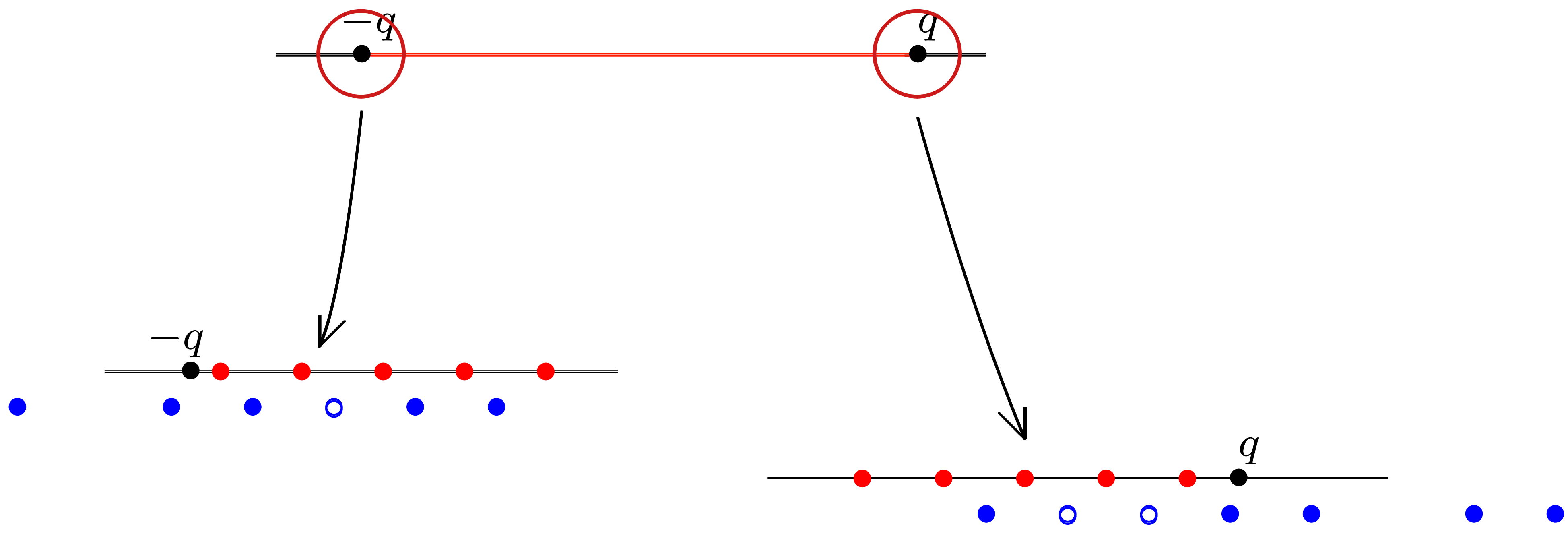
$$\mathcal{E}_{ex} = 0, \qquad \mathcal{P}_{ex} = 0$$



$$\mathcal{E}_{ex} = O(1/L), \qquad \mathcal{P}_{ex} = O(1/L)$$



$$\mathcal{E}_{ex} = O(1/L), \qquad \mathcal{P}_{ex} = O(1/L)$$



$$\mathcal{E}_{ex} = O(1/L), \qquad \mathcal{P}_{ex} = O(1/L)$$