Flat spectrum of cosmological density perturbations from conformal invariance

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Introduction

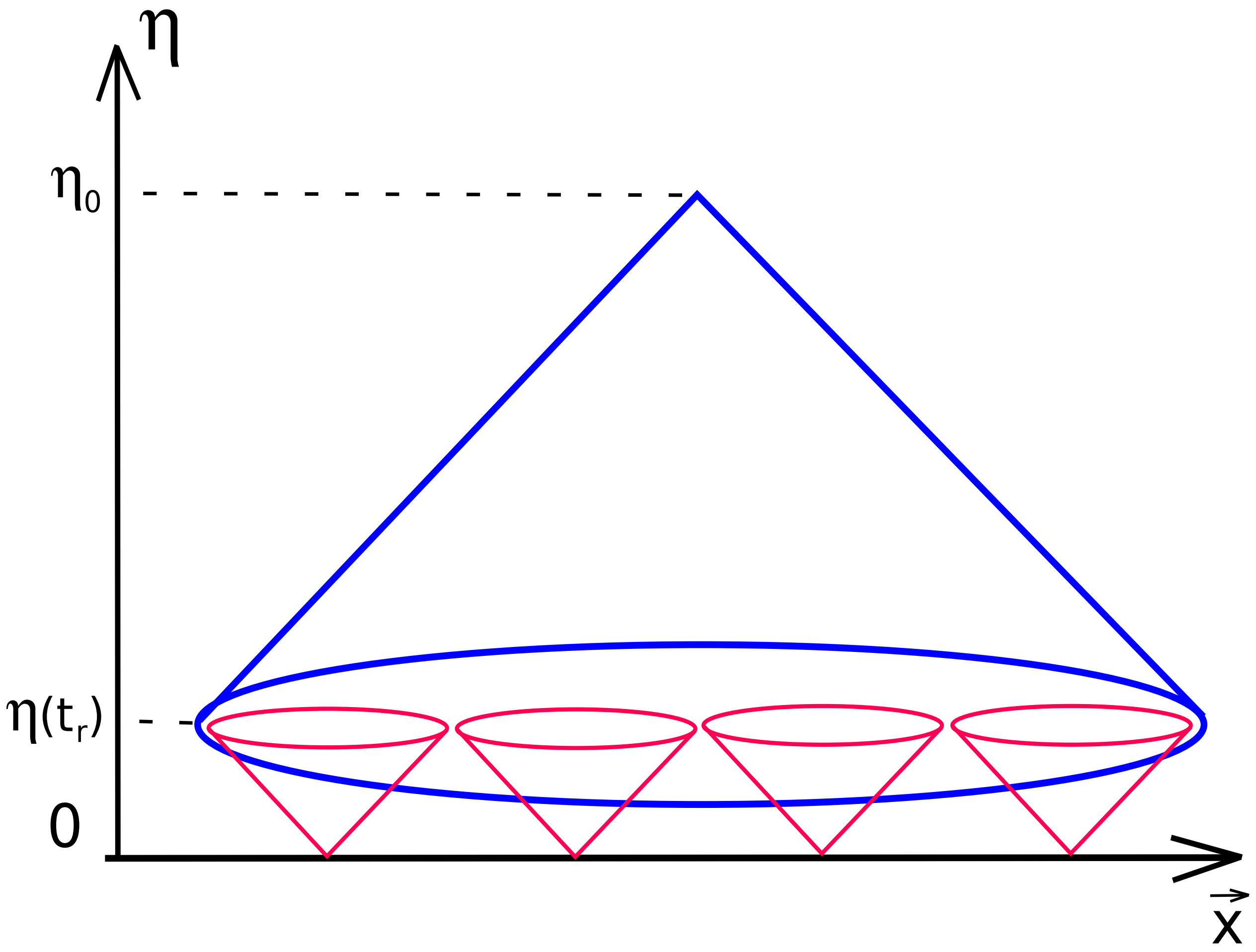
Primordial perturbations: key issue in theory of the early Universe

- Density perturbations (scalar modes),
  Discovered long ago
- Gravity waves (tensor modes)
  Not discovered (yet?)

Origin: before hot expansion epoch

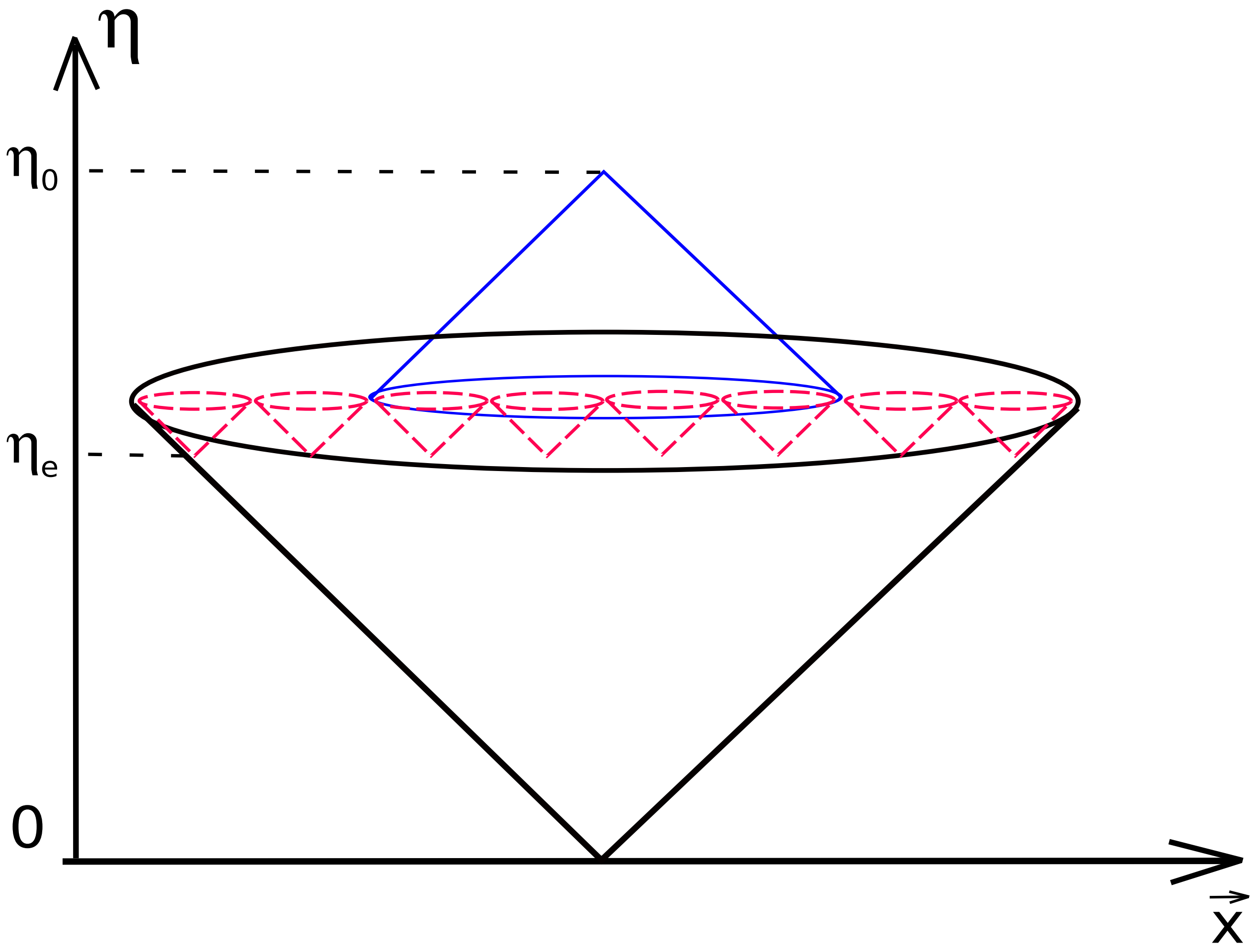
Perturbations of cosmologically relevant sizes (say, 1 Mpc) were superhorizon at hot expansion epoch

Hot expansion theory: horizon problem



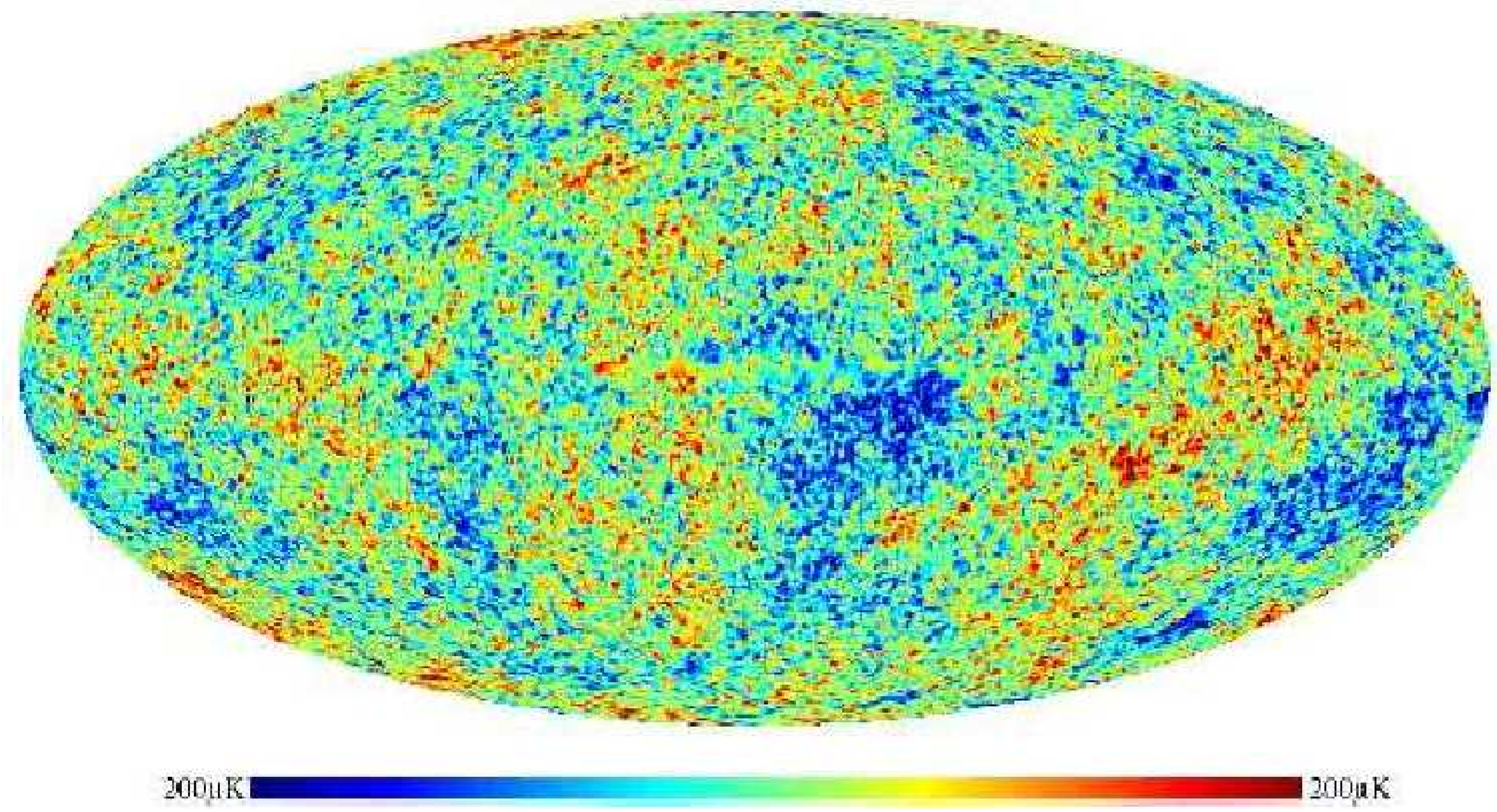
$$ds^2 = a^2(\eta)(d\eta^2 - d\mathbf{x}^2)$$

Pre-hot epoch must solve horizon problem



Perturbations had been sub-horizon at pre-hot epoch, and then became longer than the Hubble size.

Properties of primordial perturbations



Gaussian random field

Gaussian random density perturbations

- Completely determined by two-point correlator
- Higher correlators obey the Wick theorem

Hint towards the origin:

Enhanced vacuum fluctuations of free quantum field(s)

- Primordial power spectrum (almost) flat (Harrison–Zeldovich)

$$\langle \delta\rho(\mathbf{k})\delta\rho^*(\mathbf{k}')\rangle = P(k)\delta(\mathbf{k}-\mathbf{k}')$$

$$P(k) \propto k^{-3} \quad \Longleftrightarrow \quad \langle [\delta\rho(\mathbf{x})]^2\rangle = \int \mathrm{const}\cdot\frac{dk}{k}$$

Generation of primordial perturbations

- Candidate theory: inflation

$$ds^2 = dt^2 - e^{2Ht} d\mathbf{x}^2$$

  $H \approx \text{const}$

  Symmetry under spatial dilatations + time translations $\Longrightarrow$

  flat power spectrum of field fluctuations $\Longrightarrow$

  flat power spectrum of $\delta\rho$

- Any competitors?

Ekpyrotic/cycling/pre-Big Bang scenarios:

- contracting Universe $\Longrightarrow$ bounce $\Longrightarrow$ expansion

...; Veneziano;
Khouri, Ovrut, Steinhardt, Turok;
Creminelli, Luty, Nicolis, Senatore; ...

- Flat spectrum of density perturbations:

  Scalar fields with negative exponential potentials

Finelli;
Lehners, McFadden, Turok, Steinhardt;
Creminelli, Senatore;
E.Buchbinder, Khouri, Ouvrut

Problematic: fine tuning of initial conditions

Conformal + global symmetry at pre-hot epoch

Conformal scalar theory with negative quartic potential

$$S = \int \sqrt{-g} d^4x \left[g^{\mu\nu} \partial_\mu \phi^* \partial_\nu \phi + \frac{1}{6} R |\phi|^2 - V(\phi) \right]$$

$$V(\phi) = -\lambda |\phi|^4$$

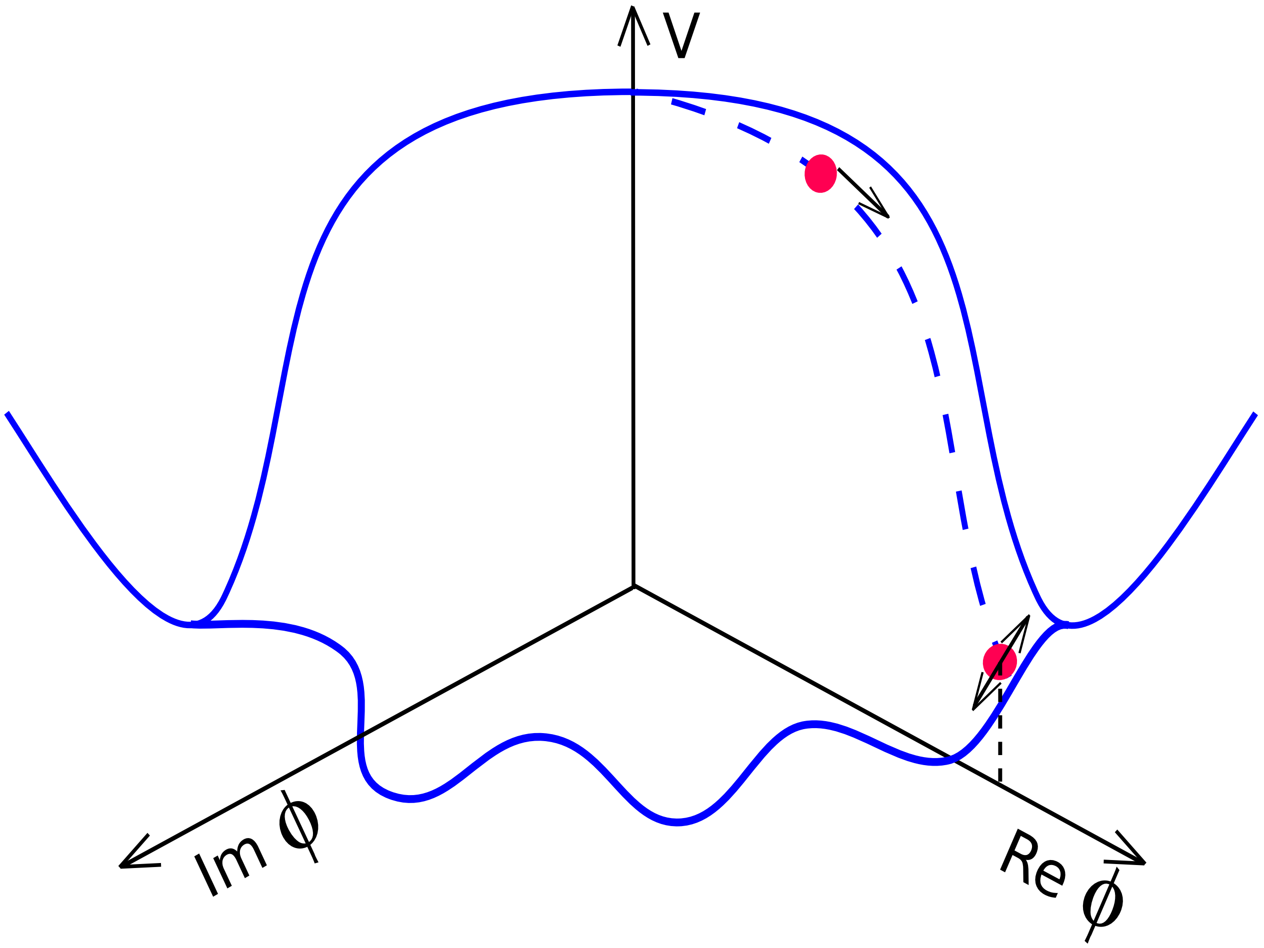
Conformal symmetry at classical level,

global symmetry $\phi \to \mathrm{e}^{i\alpha} \phi$

$\phi = \chi(\eta, \mathbf{x})/a(\eta) \Longrightarrow$ Minkowski action in conformal coordinates

$$S = \int d^3x d\eta \; [\eta^{\mu\nu} \partial_\mu \chi^* \partial_\nu \chi - V(\chi)]$$

Rolling scalar field



As the field rolls down, its phase
– Classically freezes out
– Develops quantum fluctuations with flat power spectrum

Spatially homogeneous classical evolution

- One of the classical field equations: global charge conservation. $\chi = \rho e^{i\theta}$

$$\frac{d}{d\eta}\left(\rho^2 \frac{d\theta}{d\eta}\right) = 0$$

As ρ rolls down, θ freezes out, $\theta \to \text{const} = 0$ for convenience.

- Classical solution for $\rho \equiv \text{Re}\chi \equiv \chi_c$

$$\chi_c = \frac{1}{\sqrt{\lambda}(\eta_* - \eta)}$$

Dictated by conformal invariance

Imaginary part $\propto$ phase: quantum field

$\chi = \chi_c(\eta) + \delta\hat{\chi}(\mathbf{x}, \eta)$; $\quad \hat{\chi}_2 = \mathrm{Im}\delta\hat{\chi} \Longleftrightarrow$ phase
Linearized equation in momenum representation

$$\frac{\partial^2}{\partial\eta^2}\hat{\chi}_2 + k^2\,\hat{\chi}_2 - 2\lambda\,\chi_c^2(\eta)\,\hat{\chi}_2 = 0$$

Early times, $k(\eta_* - \eta) \gg 1$: $\sqrt{\lambda}\chi_c(\eta)$ small; χ_2 is free massless field,

$$\hat{\chi}_2 = \frac{1}{(2\pi)^{3/2}\sqrt{k}}\left[\mathrm{e}^{ik\eta}A^\dagger_{\mathbf{k}} + \mathrm{e}^{-ik\eta}A_{\mathbf{k}}\right]$$

Late times, $k(\eta_* - \eta) \ll 1$: k negligible,

$$\hat{\chi}_2 = \frac{\hat{c}_{\mathbf{k}}}{k(\eta_* - \eta)}$$

Same behavior as $\chi_c(\eta)$

$$\hat{\chi}_2 = \frac{\hat{c}_{\mathbf{k}}}{k(\eta_* - \eta)}$$

Dictated by global $U(1)$ symmetry. Momentum negligible $\Longrightarrow$ homogeneous solution.

$e^{i\alpha}\chi_c(\eta)$ is a solution $\Longrightarrow$ $i\alpha\chi_c(\eta)$ is a solution to linearized eqn.

$\Longrightarrow \chi_2 \propto \chi_c(\eta)$

k^{-1} on dimensional grounds.

$\hat{c}_{\mathbf{k}}$ from matching at $k(\eta_* - \eta) \sim 1$:

$$\hat{c}_{\mathbf{k}} = \frac{\#}{\sqrt{k}}\left[e^{ik\eta_*}A^{\dagger}_{\mathbf{k}} + e^{-ik\eta_*}A_{\mathbf{k}}\right]$$

Hence,

$$\hat{\chi}_2 = \frac{\#}{k^{3/2}(\eta_* - \eta)}\left[e^{ik\eta_*}A^{\dagger}_{\mathbf{k}} + e^{-ik\eta_*}A_{\mathbf{k}}\right]$$

Phase field

$$\hat{\theta}(\mathbf{k}) = \frac{\hat{\chi}_2}{\chi_c(\eta)} = \# \frac{\sqrt{\lambda}}{k^{3/2}} \left[\mathrm{e}^{ik\eta_*} A^\dagger_{\mathbf{k}} + \mathrm{e}^{-ik\eta_*} A_{\mathbf{k}} \right]$$

Gaussian random field with flat power spectrum,

$$\langle \hat{\theta}(\mathbf{k}) \hat{\theta}(\mathbf{k}') \rangle = \# \frac{\lambda}{k^3}$$

NB: Need long period of rolling down

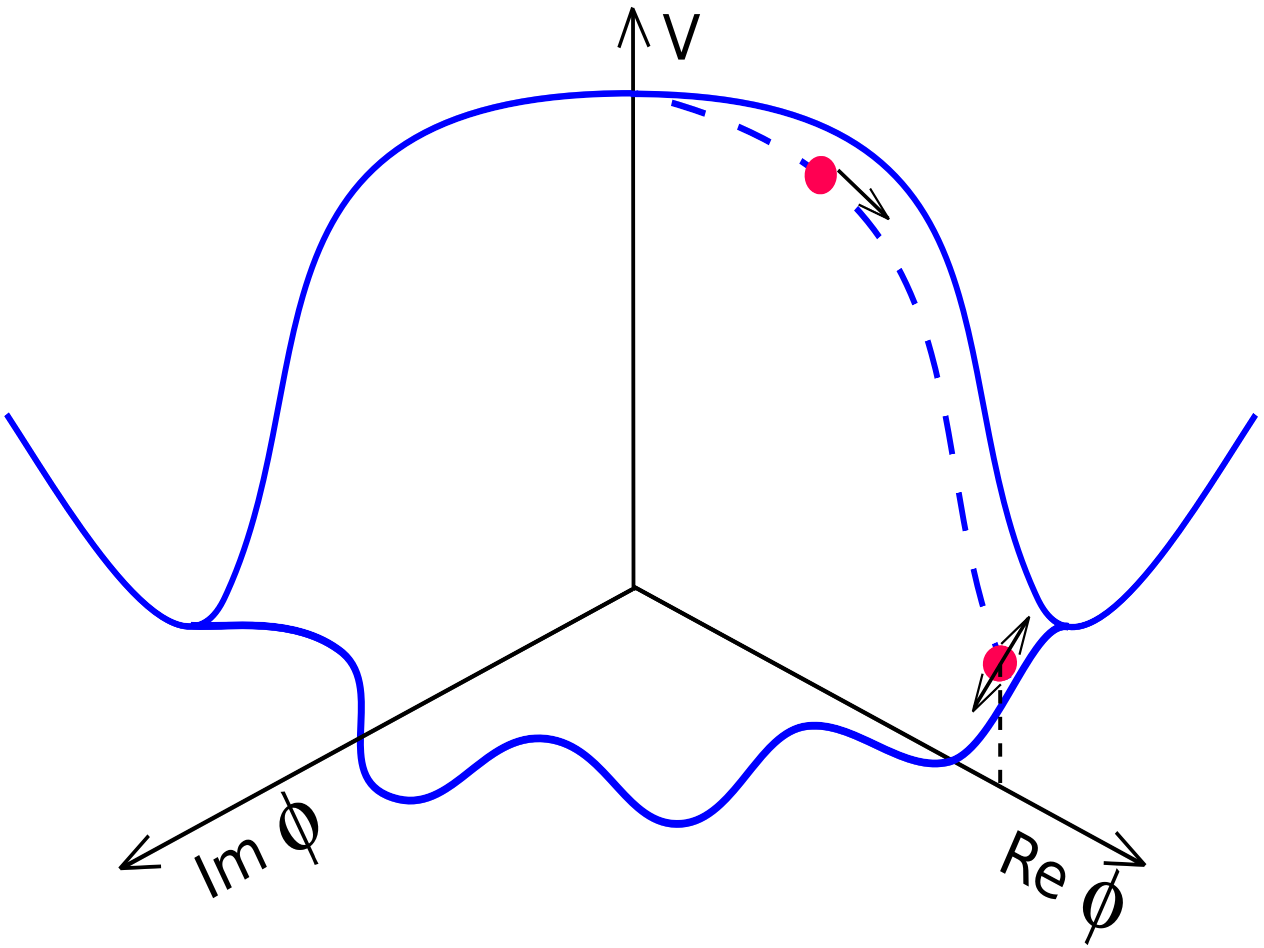
$$k(\eta_* - \eta) \ll 1 \quad \text{for cosmological present wavelength}$$

Possible in theories that solve horizon problem

Small $\lambda \Longrightarrow$ small amplitude of perturbations (similar to inflation).

Reprocessing into density perturbations

Phase = pseudo-Goldstone field a la axion = curvaton



Phase ends up on a slope of its potential.
Phase starts to oscillate at hot epoch. Its perturbations are perturbations in energy density.

To conclude

- As simple mechanism as inflationary
- Can work in any cosmological model that solves (at least formally) horizon problem
- Possible (but not necessary) sizeable non-Gaussianity, like in other curvaton models
- Small breaking of conformal invariance $\Longrightarrow$ small tilt in power spectrum (?)
- Does not imply primordial gravity waves with (almost) flat power specrum, unlike the simplest inflationary models

PRIMORDIAL GRAVITY WAVES (tensor modes) IS A KEY