Gravitational catalysis of dynamical symmetry breaking

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- To describe the low energy physics one needs to use the effective models
- The Nambu–Jona-Lasinio model (NJL)
  Y. Nambu and G. Jona-Lasinio, Phys. Rev. **124**, 246 (1961); **124**, 246 (1961)

(Nambu N.P. 2008)

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The existence of color superconductivity (CSC) is predicted for heavy ion collisions and in interiors of neutron stars.

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- Gross-Neveu in D=1+1: symmetry is broken even at $G \ll 1$: $G_c = 0$ in $(1+1)$

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- Magnetic field B (lowest Landau level)
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- The influence of μ and T on phase transitions

The model

The extended NJL model with interacting diquarks:

$$\mathcal{L} = \bar{q}\left[i\gamma^{\mu}\nabla_{\mu} + \mu\gamma^{0}\right]q + \frac{G_1}{2N_c}\left[(\bar{q}q)^2 + \left(\bar{q}i\gamma^5\vec{\tau}q\right)^2\right] + $$
$$+ \frac{G_2}{N_c}\left[i\bar{q}_c\varepsilon\epsilon^b\gamma^5 q\right]\left[i\bar{q}\varepsilon\epsilon^b\gamma^5 q_c\right]$$

The Lagrangian is invariant under the chiral $SU(2)_L \times SU(2)_R$ and color $SU(3)_c$ groups.

Bosonization procedure

Hubbard-Stratonovich transformation:

$$\tilde{\mathcal{L}} = \bar{q}\left[i\gamma^\mu\nabla_\mu + \mu\gamma^0\right]q - \bar{q}\left(\sigma + i\gamma^5\vec{\tau}\vec{\pi}\right)q - \frac{3}{2G_1}(\sigma^2 + \vec{\pi}^2) -$$

$$- \frac{3}{G_2}\Delta^{*b}\Delta^b - \Delta^{*b}\left[i\bar{q}^t C\varepsilon\epsilon^b\gamma^5 q\right] - \Delta^{*b}\left[i\bar{q}\varepsilon\epsilon^b\gamma^5 C\bar{q}^t\right]$$

The constrains for boson fields:

$$\Delta^b = -\frac{G_2}{3}\, i\bar{q}^t C\varepsilon\epsilon^b\gamma^5 q, \qquad \sigma = -\frac{G_1}{3}\,\bar{q}q, \qquad \vec{\pi} = -\frac{G_1}{3}\,\bar{q}i\gamma^5\vec{\tau}q$$

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Symmetry breaking:

- $\langle\sigma\rangle \neq 0 \longrightarrow$ chiral symmetry breaking

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Symmetry breaking:

- $\langle\sigma\rangle \neq 0 \longrightarrow$ chiral symmetry breaking
- $\langle\Delta^b\rangle \neq 0 \longrightarrow$ color symmetry breaking

The effective potential in the mean field approximation

Mean fields:

$$\sigma(x) = \langle\sigma\rangle + \tilde{\sigma}(x), \quad \vec{\pi}(x) = \langle\vec{\pi}\rangle + \tilde{\vec{\pi}}(x), \quad \Delta^b(x) = \langle\Delta^b\rangle + \tilde{\Delta}^b(x)$$

Ground state:

$$\langle\Delta^1\rangle = \langle\Delta^2\rangle = \langle\vec{\pi}\rangle = 0 \quad \text{and} \quad \langle\sigma\rangle, \langle\Delta^3\rangle \neq 0$$

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Effective potential:

$$V_{\text{eff}} = \frac{3\sigma^2}{2G_1} + \frac{3\Delta\Delta^*}{G_2} + \tilde{V}_{\text{eff}}; \quad \tilde{V}_{\text{eff}} = -\frac{S_q}{v}, \quad v = \int d^4x\sqrt{-g}$$

$$\begin{aligned} S_q(\sigma,\Delta) = \; & - \; \mathrm{i}\ln\mathrm{Det}\left[\mathrm{i}\hat{\nabla} - \sigma + \mu\gamma^0\right] \\ & - \; \tfrac{\mathrm{i}}{2}\ln\mathrm{Det}\left[4|\Delta|^2 + (-\mathrm{i}\hat{\nabla} - \sigma + \mu\gamma^0)(\mathrm{i}\hat{\nabla} - \sigma + \mu\gamma^0)\right] \end{aligned}$$

Hyperbolic space $R \otimes H^3$

The metrics of hyperbolic space:

$$ds^2 = dt^2 - a^2(d\theta^2 + \sinh^2\theta d\Omega_2)$$

a – the radius of the hyperboloid,
$d\Omega_2$ – the metrics on unit 2-sphere

The scalar curvature

$$R = -\frac{6}{a^2} < 0$$

γ-matrices, covariant derivative and spin connection:

$$\{\gamma_\mu(x), \gamma_\nu(x)\} = 2g_{\mu\nu}(x),\ \{\gamma_{\hat{a}}, \gamma_{\hat{b}}\} = 2\eta_{\hat{a}\hat{b}},\ \eta_{\hat{a}\hat{b}} = \mathrm{diag}(1, -1, -1, -1$$

$$g_{\mu\nu}g^{\nu\rho} = \delta^\rho_\mu, \quad g^{\mu\nu}(x) = e^\mu_{\hat{a}}(x)e^{\nu\hat{a}}(x), \quad \gamma_\mu(x) = e^{\hat{a}}_\mu(x)\gamma_{\hat{a}}.$$

$$\nabla_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\omega^{\hat{a}\hat{b}}_\mu \sigma_{\hat{a}\hat{b}}, \quad \sigma_{\hat{a}\hat{b}} = \frac{1}{4}[\gamma_{\hat{a}}, \gamma_{\hat{b}}],$$

$$\omega^{\hat{a}\hat{b}}_\mu = \frac{1}{2}e^{\hat{a}\lambda}e^{\hat{b}\rho}[C_{\lambda\rho\mu} - C_{\rho\lambda\mu} - C_{\mu\lambda\rho}], \quad C_{\lambda\rho\mu} = e^{\hat{a}}_\lambda \partial_{[\rho}e_{\mu]\hat{a}}.$$

Thermodynamic potential and gap equations

The regularized effective potential at finite temperature or thermodynamic potential (TDP):

$$
\begin{aligned}
\Omega^{\mathrm{reg}}(\sigma,\Delta) &= 3\left(\frac{\sigma^2}{2G_1}+\frac{|\Delta|^2}{G_2}\right) \\
&- \frac{2}{\pi^2}\int_0^\Lambda dp(p^2+\frac{|R|}{24})\{E_p + T\ln\left(1+e^{-\beta(E_p\pm\mu)}\right) \\
&+ \sqrt{(E_p\pm\mu)^2+4|\Delta|^2}+2T\ln\left(1+e^{-\beta\sqrt{(E_p\pm\mu)^2+4|\Delta|^2}}\right)\}
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The global minimum point of TDP is determined from the gap equations:

$$\frac{\partial\Omega^{\mathrm{reg}}}{\partial\sigma}=0,\qquad \frac{\partial\Omega^{\mathrm{reg}}}{\partial|\Delta|}=0$$

Flat case

$\sigma \neq 0,\ \Delta = 0 \quad \mu = 0,\ T = 0$:

Dimensionless quantities: $x = \frac{\sigma}{\Lambda}, \quad g = \frac{\Lambda^2}{\pi^2} G_1, \quad r = \frac{R}{\Lambda^2}$

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The Gap equation:

$$\frac{1}{g} = \sqrt{1+x^2} - x^2 \ln \frac{1+\sqrt{1+x^2}}{x}$$

The solution exists only at $g > g_c = 1$

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Gravitational catalysis of symmetry breaking

If $\sigma \ll \Lambda$, $\sigma^2 \ll \frac{|R|}{12}$:

$$\sigma = 2\Lambda \exp\left[-\frac{12}{|r|}\left(\frac{1}{g} - 1\right)\right] = 2\Lambda \exp\left[-\frac{12\pi^2(1-g)}{|R| G_1}\right]$$

- Magnetic field

$$\sigma = \sqrt{\frac{|eB|}{\pi}} \exp\left[-\frac{2\pi^2(1-g)}{|eB| G_1}\right],$$

The comparison between gravitational and magnetic fields

- Magnetic field

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- Gravitational field

$$\sigma = 2\Lambda \exp\left[-\frac{12\pi^2(1-g)}{|R|\, G_1}\right]$$

The growing $|R|$ leads to increasing of σ.

The mixed phase

$\sigma \neq 0,\ \Delta \neq 0,\ A = \frac{1}{g} = \frac{\pi^2}{\Lambda^2 G_1},\ B = \frac{3\pi^2}{4\Lambda^2 G_2},\ A \leq B < 1$:

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- Zero temperature:

$$\sigma(0) = 2\Lambda \exp\left[-\frac{12}{|r|}(3A - 2B - 1)\right],$$

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- Finite temperature:

$$\begin{aligned} \sigma(T) &= \sigma(0) \exp\left[-\mathrm{I}_1(\beta\sigma(T))\right], \\ m_*(T) &= m_*(0) \exp\left[-\mathrm{I}_1(\beta m_*(T))\right] \end{aligned}$$

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- Critical temperatures:

$$T_c^\sigma = \pi^{-1} e^C \sigma(0), \qquad T_c^\Delta = \pi^{-1} e^C m_*(0)$$

The dependence of condensates on temperature

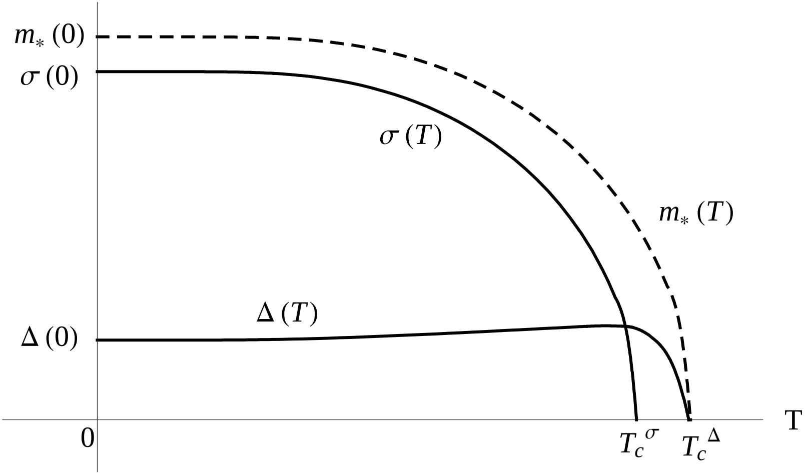

Figure: Condensates σ, Δ and m_* as functions of temperature inside the mixed phase.

Phase transitions at $\mu \neq 0$

$$G_2 = \frac{3}{8} G_1$$

$g = G_1 \Lambda^2 / \pi^2 = 1.404 \longrightarrow \sigma = 350$ MeV at $R = 0$.

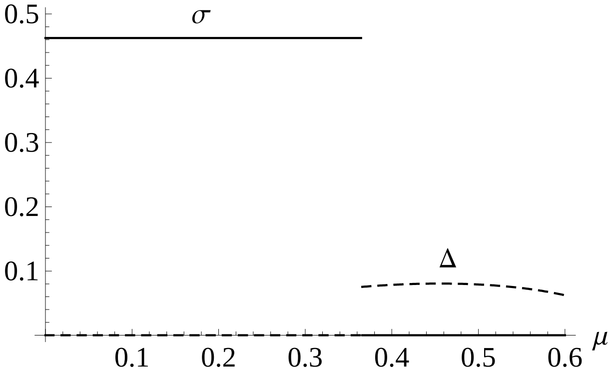

Figure: Condensates σ and Δ as functions of μ at $|r| = 1$.

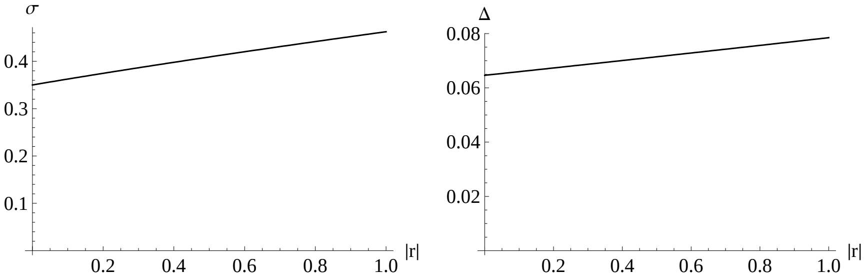

Figure: Condensates σ at $\mu = 200$ MeV (left) and Δ at $\mu = 400$ MeV (right) as functions of $|r|$.

At $g > 1$ curvature leads to an enhancement of symmetry breaking.

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Thank you for your attention!