RENORMALIZABLE LORENTZ INVARIANT FORMULATION OF THE YANG-MILLS THEORY WITHOUT GRIBOV AMBIGUITY.

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A problem of unambiguos quantization of nonabelian gauge theories beyond perturbation theory remains unsolved.

Differential gauge conditions: $L(A_\mu, \varphi) = 0 \rightarrow$ **Gribov ambiguity.**

Algebraic gauge conditions: $\tilde{L}(A_\mu, \varphi) = 0 \rightarrow$ **absence of the manifest Lorentz invariance and other problems.**

A remedy: new formulation of the Yang-Mills theory using more ghost fields.

The model is described by the classical ($SU(2)$)Lagrangian

$$
\begin{aligned}
L = -\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + (D_\mu\varphi)^*(D_\mu\varphi) - (D_\mu\chi)^*(D_\mu\chi) \\
-g^{-1}[(D_\mu\varphi)^* + (D_\mu\chi)^*](D_\mu\widehat{m}) - g^{-1}(D_\mu\widehat{m})^*[D_\mu\varphi + D_\mu\chi] \\
+i[(D_\mu b)^*(D_\mu e) - (D_\mu e)^*(D_\mu b)]
\end{aligned}
\qquad (1)
$$

The scalar fields (φ, χ are commuting, e, b are anticommuting) are parametrized by the Hermitean components

$$
\Phi = \left(\frac{i\Phi_1 + \Phi_2}{\sqrt{2}}, \frac{\Phi_0 - i\Phi_3}{\sqrt{2}}\right) \qquad (2)
$$

$\widehat{m}$ is a constant spinor.

$$\widehat{m} = (0, m) \tag{3}$$

The Lagrangian (1) may be obtained from the gauge invariant Lagrangian, describing the interaction of the complex scalar doublets with the Yang-Mills field by the shift

$$\varphi \to \varphi - g^{-1}\widehat{m}; \quad \chi \to \chi + g^{-1}\widehat{m} \tag{4}$$

Hence the Lagrangian (1) is invariant with respect to the"shifted"gauge transformations.

In particular the transformation of the field $\varphi_-^a = \frac{\varphi - \chi}{\sqrt{2}}$ is
$\delta\varphi_-^a = m\eta^a + \frac{g}{2}\varepsilon^{abc}\varphi_-^b\eta^c + \frac{g}{2}\varphi_-^0\eta^a$

This Lagrangian is also invariant with respect to the supersymmetry transformations

$$\begin{aligned} \delta\varphi_\alpha^-(x) &= 2i\epsilon b_\alpha(x) \\ \delta e_\alpha(x) &= \epsilon\varphi_\alpha^+(x) \\ \delta b(x) &= 0 \end{aligned} \tag{5}$$

where ϵ is a constant anticommuting parameter.

This invariance plays a crucial role in the proof of the equivalence of the model described by the Lagrangian (1) to the standard Yang-Mills theory.It provides the unitarity of the scattering matrix in the subspace which includes only three dimensionally transversal components of the Yang-Mils field.

The field φ_-^a is shifted under the gauge transformation by an arbitrary function $m\eta^a$. It allows to impose Lorentz invariant algebraic gauge condition $\varphi_-^a = 0$ and hence avoid the Gribov ambiguity.

A canonical quantization in the gauge $\varphi_-^a = 0$ requires introduction of ultralocal ghosts. So the gauge fixing is introduced by adding to the action the term

$$s\int d^4x \bar{c}^a \varphi_-^a = \int d^4x(\lambda^a \varphi_-^a - \bar{c}^a M_{ab} c_b) \tag{6}$$

$$M_{ab} = \delta_{ab}(m + \frac{g}{2}\varphi_-^0) + \frac{g}{2}\varepsilon_{abc}\varphi_-^b \tag{7}$$

Imposing the gauge condition $\varphi_-^a = 0$ we break the invariance of the effective action with respect to the supersymmetry transformation (5).

However, as the transition from one gauge to the other one may be achieved by a gauge transformation, and in the gauge $\partial_i A_i = 0$ the effective action is invariant with respect to the supertransformation (5), in the gauge $\varphi_-^a = 0$ it also must be invariant with respect to some supertransformation. The corresponding gauge function is a solution of the equation

$$\int d^4x \lambda^a(x) \partial_i (A^\Omega)_i^a(x) = \int d^4x \lambda^a(x) \varphi_-^a(x) \qquad (8)$$

The invariance with respect to the supertransformation(5) and the BRST transformations corresponding to the gauge invariant classical action in the gauge $\varphi_-^a = 0$ allows to find the exact solution of the equation (8)and therefore the transformation which leaves invariant the effective action in the gauge $\varphi_-^a = 0$:

$$S_{ef} = \int dx\{-\frac{1}{4}F_{\mu\nu}^a F_{\mu\nu}^a + \partial_\mu\varphi_+^0\partial_\mu\varphi_-^0 + m\partial_\mu\varphi_+^a A_\mu^a + \frac{g^2}{4}A_\mu^2\varphi_+^0\partial_\mu\varphi_-^0 + \frac{mg}{2}A_\mu^2\varphi_+^0 + \frac{g}{2}\partial_\mu\varphi_-^0\varphi_+^a A_\mu^a - \frac{g}{2}\varphi_-^0\partial_\mu\varphi_+^a A_\mu^a + \lambda^a\varphi_-^a + i[(D_\mu b)^* D_\mu e - D_\mu e)^* D_\mu b]\} \quad (9)$$

The spectrum:

Ghost exitations: $\varphi_{\pm}, b, e,$ longitudinal and temporal components of A_{μ}^{a}
Physical exitations: three dimensionally transversal components of the Yang-Mills field.

The supersymmetry of the effective action generates a conserved nilpotent charge Q. Physical states are separated by the condition

$$Q|\psi >_{ph} = 0 \tag{10}$$

By the standard methods one can show that the states separated by this condition describe only three dimensionally transversal components of the Yang-Mills field.

The ghost exitations decouple.

Renormalization

The propagator $\varphi^a_+ A^b_\mu \sim k_\mu k^{-2} \rightarrow$ divergent diagrams with arbitrary number of external φ^0_- lines. Index of divergency

$$n = 4 - 2L_{\varphi^0_+} - 2L_{\varphi^0_-} - L_A - L_e - L_b \qquad (11)$$

Index of divergency ≤ 2. Renormalizability?

Change of variables:

$$\varphi^0_- = \frac{2m}{g}[\exp(\frac{gh}{2m}) - 1]; \quad \varphi^a_- = \tilde{M}\tilde{\varphi}^a_-$$
$$\varphi^a_+ = \tilde{M}^{-1}\tilde{\varphi}^a_+; \quad \varphi^0_+ = \tilde{M}^{-1}\tilde{\varphi}^0_+$$
$$e = \tilde{M}^{-1}\tilde{e}$$
$$b = \tilde{M}^{-1}\tilde{b}; \quad \lambda^a = \tilde{M}^{-1}\tilde{\lambda}^a; \quad \tilde{M} = 1 + \frac{g}{2m}\varphi^0_- \tag{12}$$

In terms of the new variables

$$S_{ef} = \int dx\{-\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} - (\partial^2 h + \frac{g}{2m}\partial_\mu h\partial_\mu h)\partial_\mu\tilde{\varphi}^0_+ - m\partial_\mu\tilde{\varphi}^a_+ A^a_\mu + \\ + \frac{mg}{2}A^2_\mu\varphi^0_+ + g\tilde{\varphi}^a_+ A^a_\mu\partial_\mu h + i[((D_\mu\tilde{b})^* + \frac{g}{2m}\tilde{b}^*\partial_\mu h)(D_\mu\tilde{e} - \frac{g}{2m}\tilde{e}\partial_\mu h) \\ -h.c.]\} \quad (13)$$

The field $h(\varphi^0_-)$enters interaction only with derivative $\partial_\mu h$.

Hence the divergency index of a diagram with n external $h(\varphi^0_-)$ lines decreases by $n \to$ renormalizable theory.

Does renormalization preserves the unitarity and all the symmetries of the theory?Yes

The possible counterterms may be classified on the basis of Generalised Ward Identities, associated with the symmetry, which combines the gauge invariance of the effective action and its supersymmetry.

$$\int d^4x \{ \frac{\delta \Gamma}{\delta \Phi^*(x)} \frac{\delta \Gamma}{\delta \Phi(x)} + 2i\tilde{b}^0(x) \frac{\delta \Gamma}{\delta h} \} = 0 \tag{14}$$

Φ are the fields: $A_\mu, \varphi_+^\alpha, e^\alpha, b^a$

Φ^* are the antifields introducing the variations of the fields Φ.

Additional constraints are provided by the residual global $SU(2)$ invariance

The renormalized action differs of the unrenormalized one only by the redefinition of the parameters which enter the unrenormalized effective action.

Conclusion.

A manifestly Lorentz invariant formulation of the Yang-Mills theory which allows a canonical quantization without Gribov ambiguity is possible.

The ghost field Lagrangian entering the corresponding effective action is gauge invariant. Like in QED the gauge invariance of the effective action is broken only by the gauge fixing term.

The model in the ambiguity free gauge is renormalizable, but the renormalization is highly nontrivial. It requires a redefinition of the fields entering the effective action.