

# Lattice QCD at Extreme Conditions

**International Conference  
"Gauge Fields. Yesterday, Today, Tomorrow"  
dedicated to the 70-th anniversary of  
Andrei Alekseevich Slavnov**

**M.I. Polikarpov, ITEP**

# Plan

- **Lattice QCD at finite temperature**

**DESY – ITEP – Kanazawa collaboration**

- Confining string breaking
- Deconfining phase transition

- **Strong magnetic fields in heavy ion collisions**

- Lattice simulations with magnetic fields
- Chiral Magnetic Effect [generation of the electric current of quarks directed along the magnetic field]
- Chiral symmetry breaking
- Magnetization of the vacuum
- Electric dipole moment of quark along the direction of the magnetic field

# Confining string breaking

DESY 02-135  
ITEP-LAT/2002-16  
KANAZAWA 02-26



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## Thermodynamics and heavy quark potential in $N_f = 2$ dynamical QCD \*

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G. Schierholz <sup>c</sup>, A. Slavnov <sup>d</sup>, H. Stüben <sup>e</sup>, T. Suzuki <sup>a</sup>, P. Uvarov <sup>b</sup>, A. Veselov <sup>b</sup>

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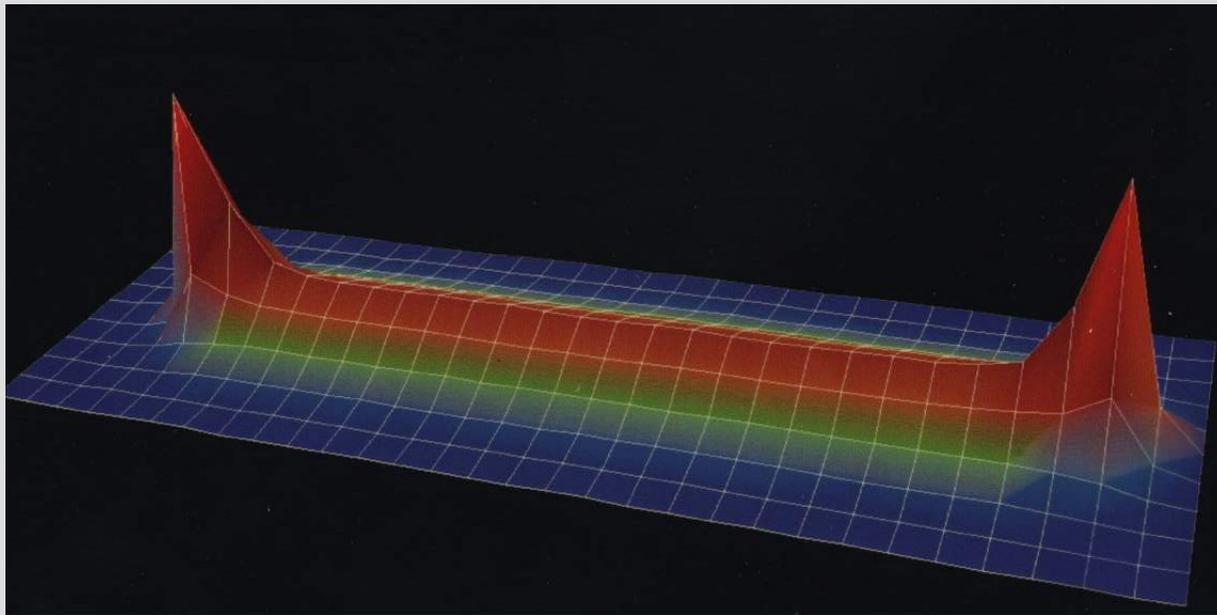
<sup>c</sup>NIC/DESY Zeuthen, Platanenallee 6, 15738 Zeuthen, Germany and  
Deutsches Elektronen-Synchrotron DESY, D-22603 Hamburg, Germany

<sup>d</sup>Steklov Mathematical Institute, Vavilova 42, 117333 Moscow, Russia

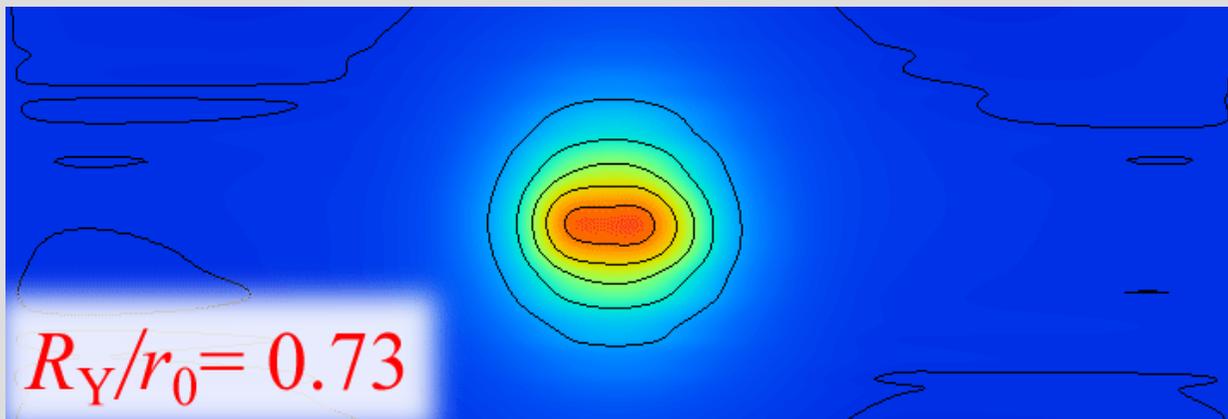
<sup>e</sup>Konrad-Zuse-Zentrum für Informationstechnik Berlin, D-14195 Berlin, Germany

We study  $N_f = 2$  lattice QCD with nonperturbatively improved Wilson fermions at finite temperature on  $16^3 \cdot 8$  lattices. We determine the transition temperature at  $\frac{m_\pi}{m_\rho} \sim 0.8$  and lattice spacing as small as  $a \sim 0.12\text{fm}$ . The string breaking at  $T < T_c$  is also studied. We find that the static potential can be fitted by a simple expression involving string model potential at finite temperature.

# Confining string breaking

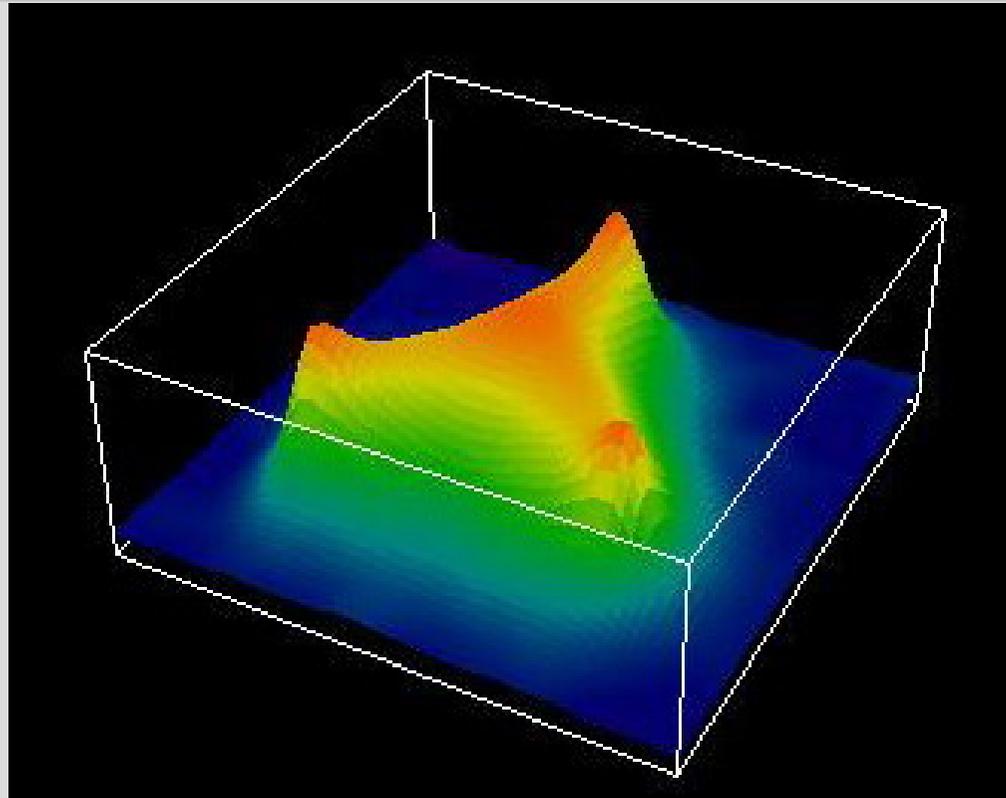


← **No dynamical quarks**



← **Full QCD**

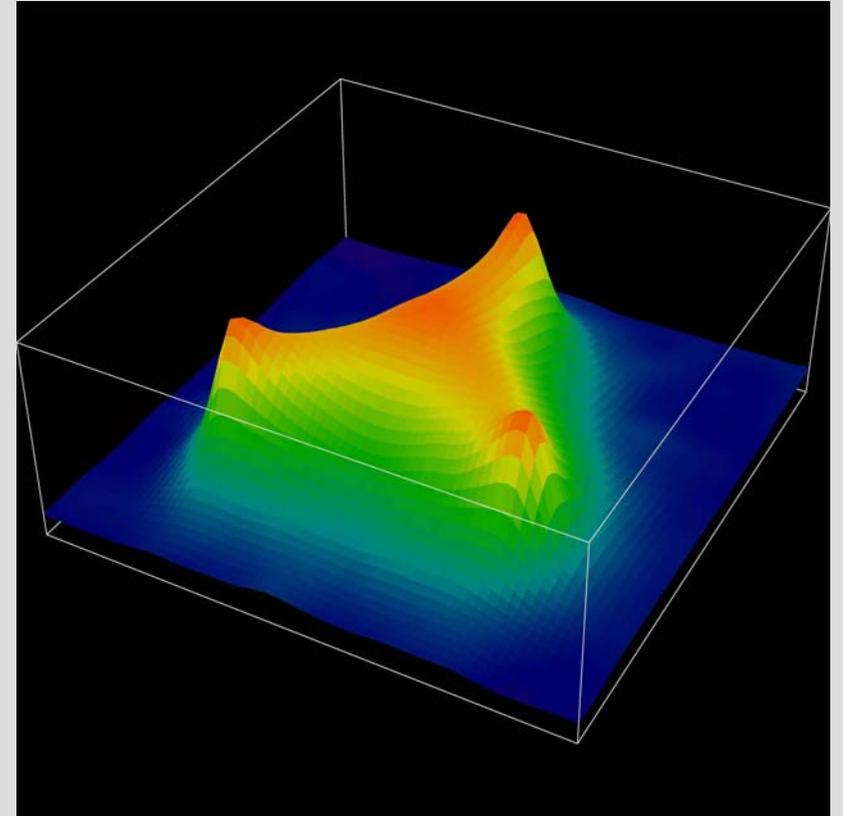
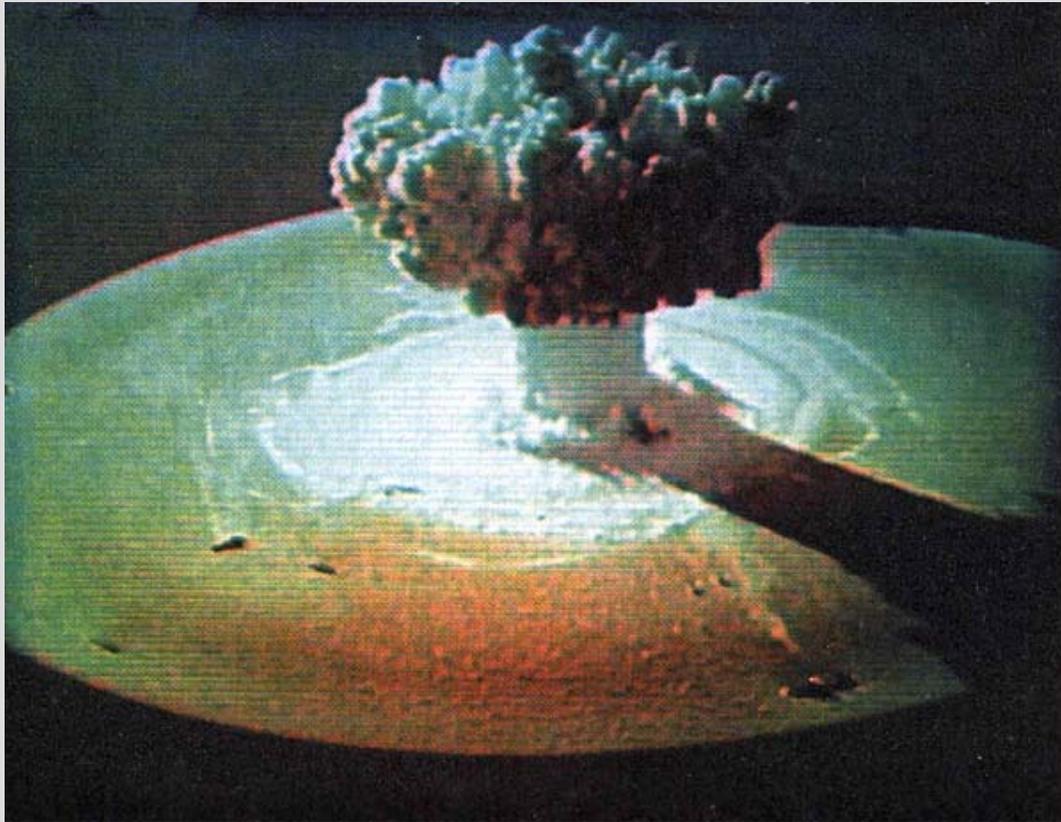
# Baryon flux tube, Delta or Y type?



**Genuine 3-body forces! (DIK collaboration results)**

$$V(r_1, r_2, r_3) \neq V(r_1 - r_2) + V(r_2 - r_3) + V(r_3 - r_1)$$

# The masses of all material objects around us are due to gluon tubes in baryons



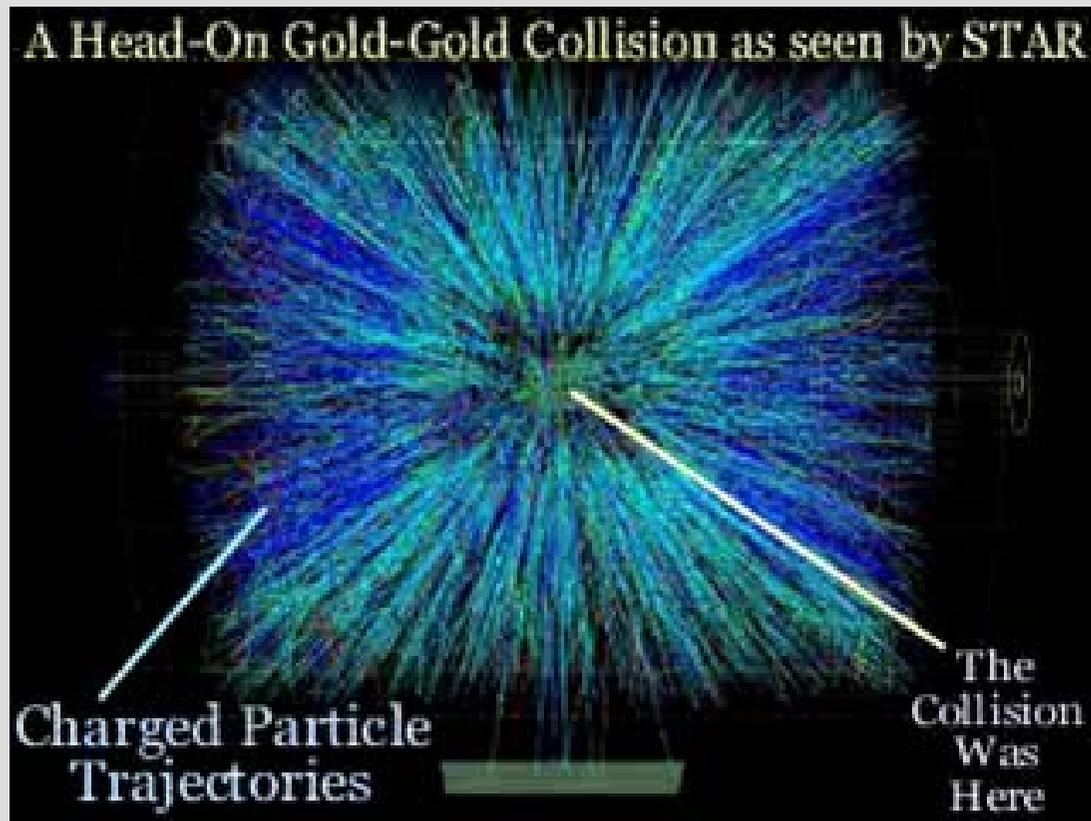
$$E = m_0 c^2$$

$$3m_q / m_{\text{baryon}} \approx 1/100$$

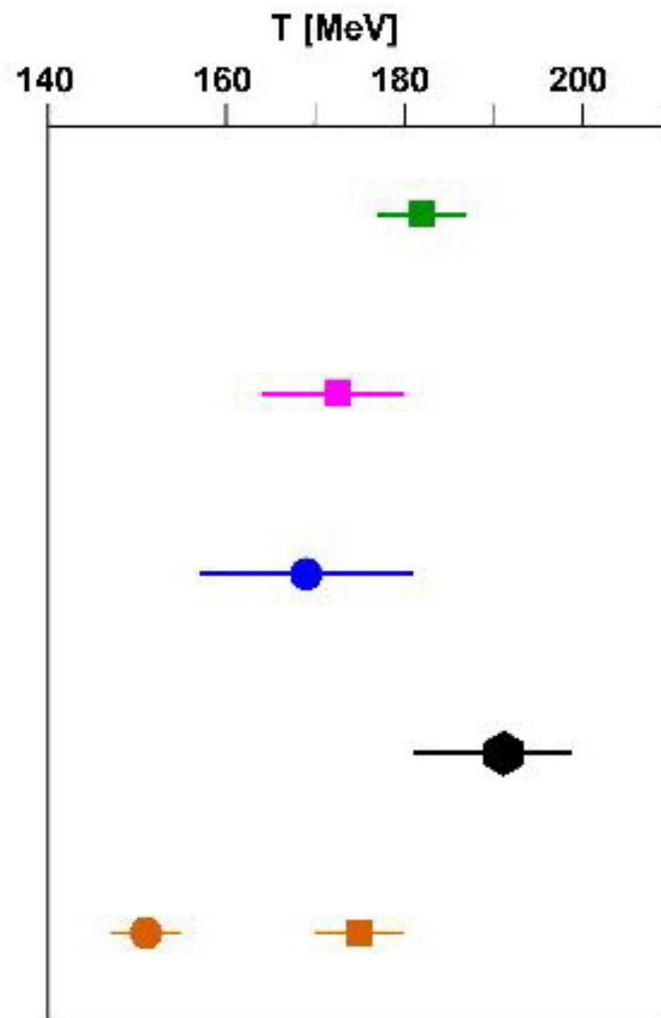
$$m_0 = \frac{E}{c^2}$$

# Critical temperature, $T_c$

**At  $T > T_c$  we have deconfining phase:  
quark-gluon plasma, glasma, quark-gluon matter**



# Summary of recent results on $T_c$



use  $T=0$  scale:  $r_0=0.469$ fm

$N_f=2$ :

V.G. Bornyakov et al, POS Lat2005, 157 (2006)

(improved Wilson,  $N_t=8, 10$ ; input:  $r_0=0.5$  fm)

(added  $N_t=12$ , Lattice'07) (rescaled to  $r_0$ )

Y. Maezawa et al., hep-lat/0702005 (QM'2006)

(improved Wilson,  $N_t=4, 6$ ; input:  $m$ -rho)

(no cont. exp. yet)

$N_f=2=1$ :

C. Bernard et al., Phys.Rev. D71, 034504 (2005)

(improved staggered (asqtad),  $N_t=4, 6, 8$ , input  $r_1$ )

(rescaled to  $r_0$ )

M. Cheng et al., Phys.Rev D74, 054507 (2006)

(improved staggered (p4),  $N_t=4, 6$ ; input  $r_0$ )

Y. Aoki et al., Phys. Lett. B643, 46 (2006)

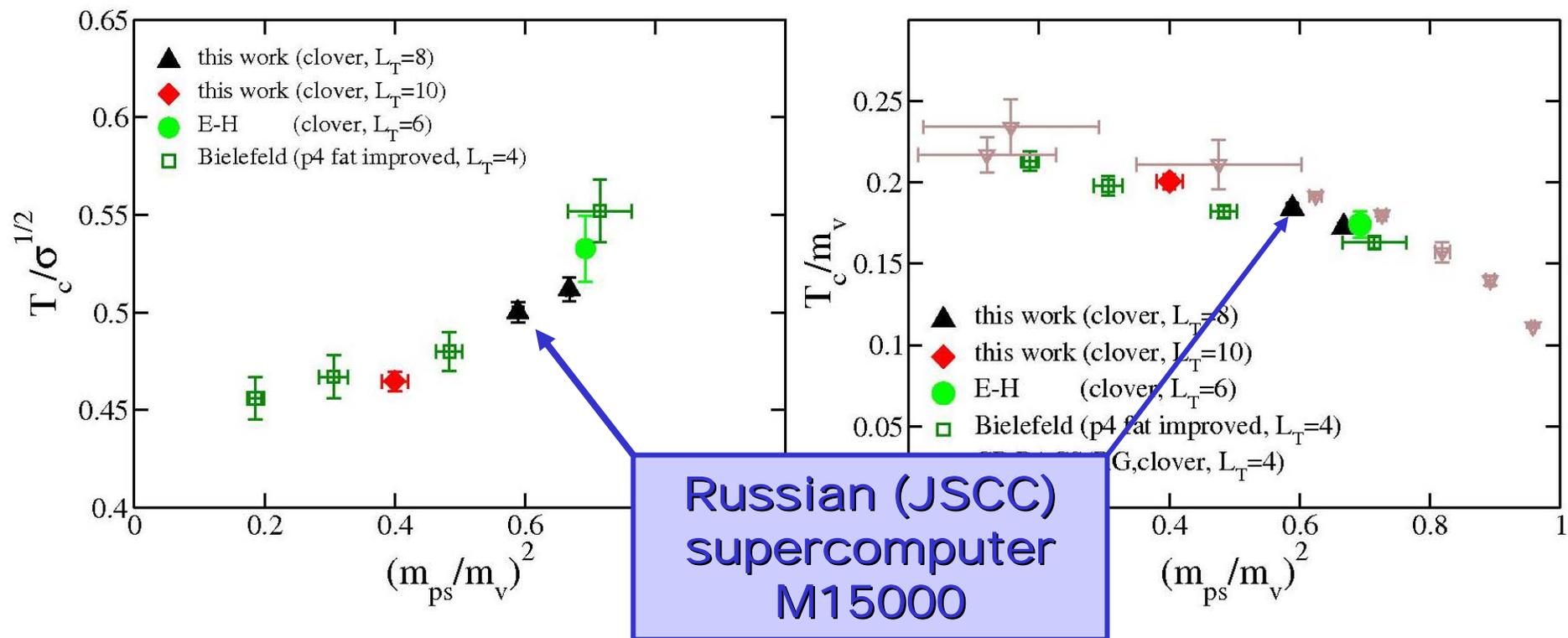
(staggered (stout),  $N_t=4, 6, 8, 10$ ; input fK)

(converted to  $r_0$ )

● chiral    ■ deconfinement    ● chiral+deconfinement

# $T_c$ by DIK (DESY-ITEP-Kanazawa) collaboration

V.G. Borneyakov, M.N. Chernodub, Y. Mori, S.M. Morozov, Y. Nakamura,  
M.I. Polikarpov, G. Schierholz, A.A. Slavnov, H. Stüben, T. Suzuki, MIP (2006)



**Figure 5:** (left) The transition temperature  $T_c$  in units of the string tension  $\sigma(T=0)$  as a function of  $m_{ps}/m_v$  and (right)  $T_c$  in units of  $m_v$  as a function of  $(m_{ps}/m_v)^2$  (right). The data is from this work (the up triangles and the diamond), Ref. [1] (the squares), Ref. [6] (the circle), and Ref. [5] (the down triangles).

# Strong Magnetic Fields in Heavy Ion Collisions

*P.V.Buividovich, M.N.Chernodub, E.V.Luschevskaya, M.I.Polikarpov*

## Comparison of magnetic fields



The Earth's magnetic field 0.6 Gauss

A common, hand-held magnet 100 Gauss



The strongest steady magnetic fields achieved so far in the laboratory  $4.5 \times 10^5$  Gauss

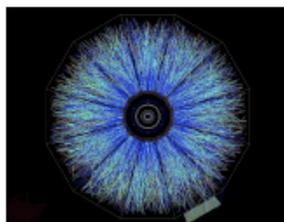
The strongest man-made fields ever achieved, if only briefly  $10^7$  Gauss



Typical surface, polar magnetic fields of radio pulsars  $10^{13}$  Gauss

Surface field of Magnetars  $10^{15}$  Gauss

<http://solomon.as.utexas.edu/~duncan/magnetar.html>



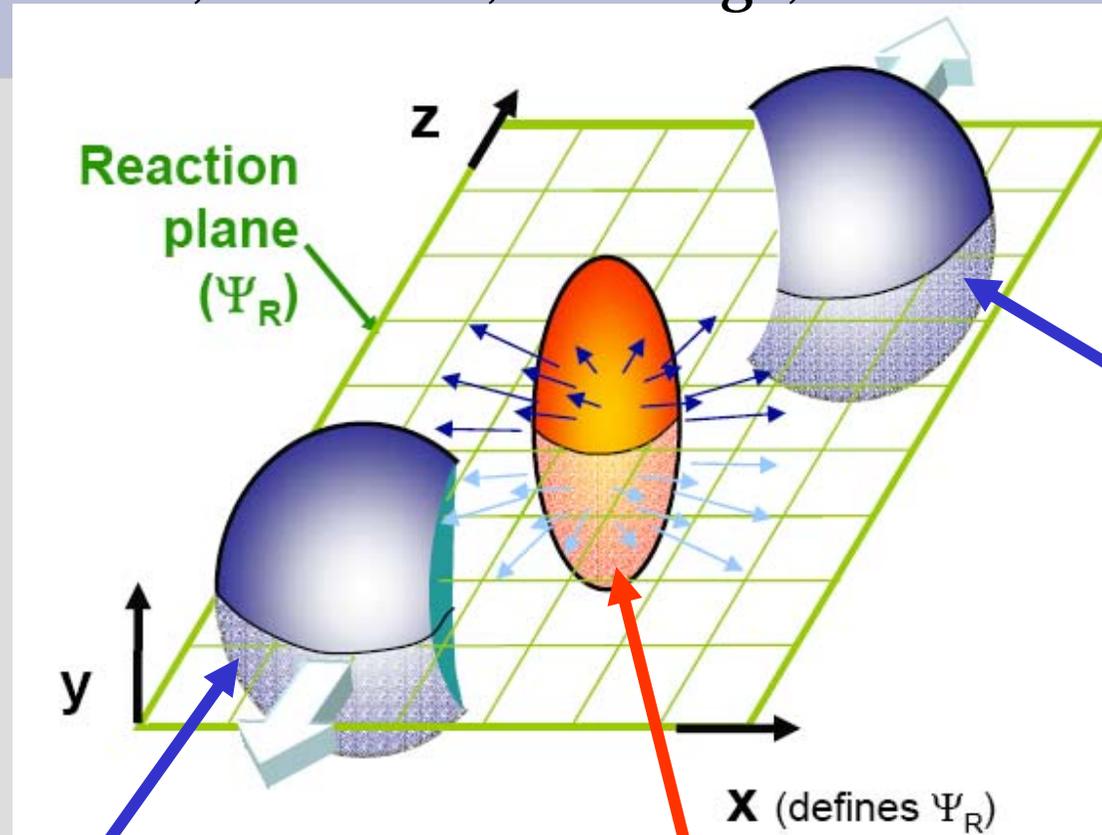
At BNL we beat them all!

Off central Gold-Gold Collisions at 100 GeV per nucleon

$$eB(\tau=0.2 \text{ fm}) = 10^3 \sim 10^4 \text{ MeV}^2 \sim 10^{17} \text{ Gauss}$$

# Magnetic fields in non-central collisions

[Fukushima, Kharzeev, Warringa, McLerran '07-'08]

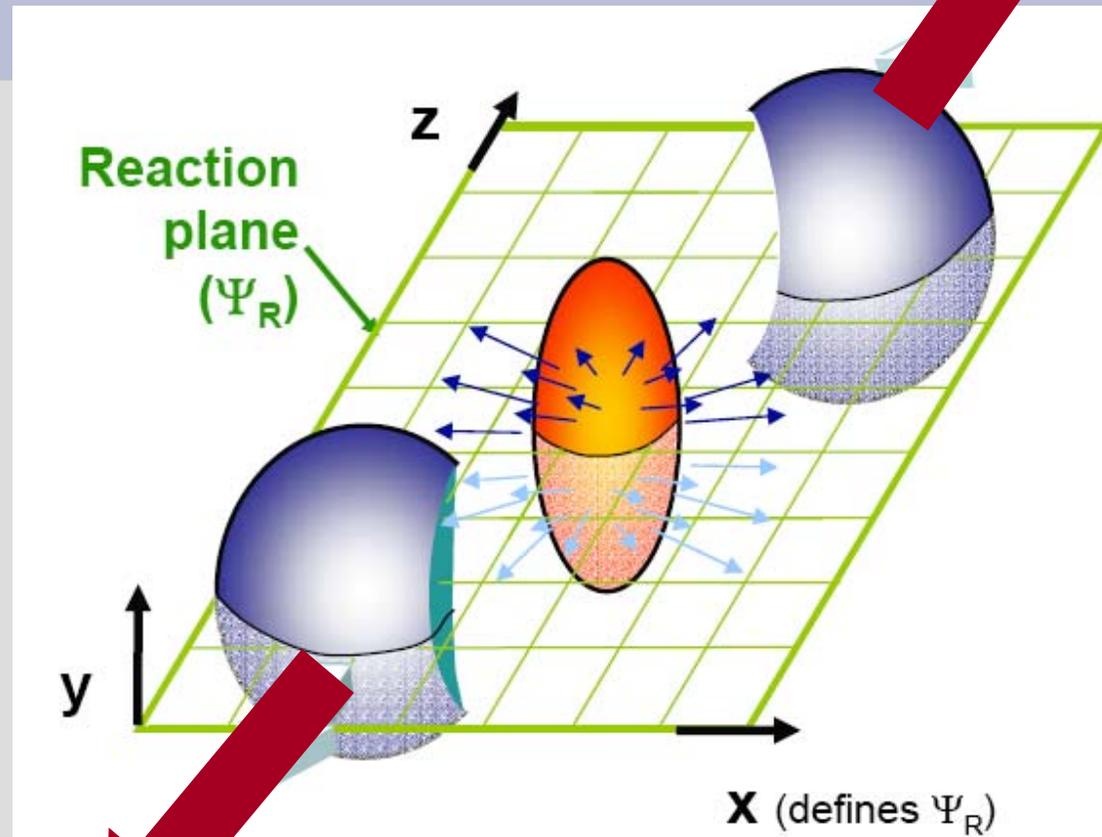


Heavy ion

Heavy ion

Quarks and gluons

# Magnetic fields in non-central collisions



Charge is large  
Velocity is high

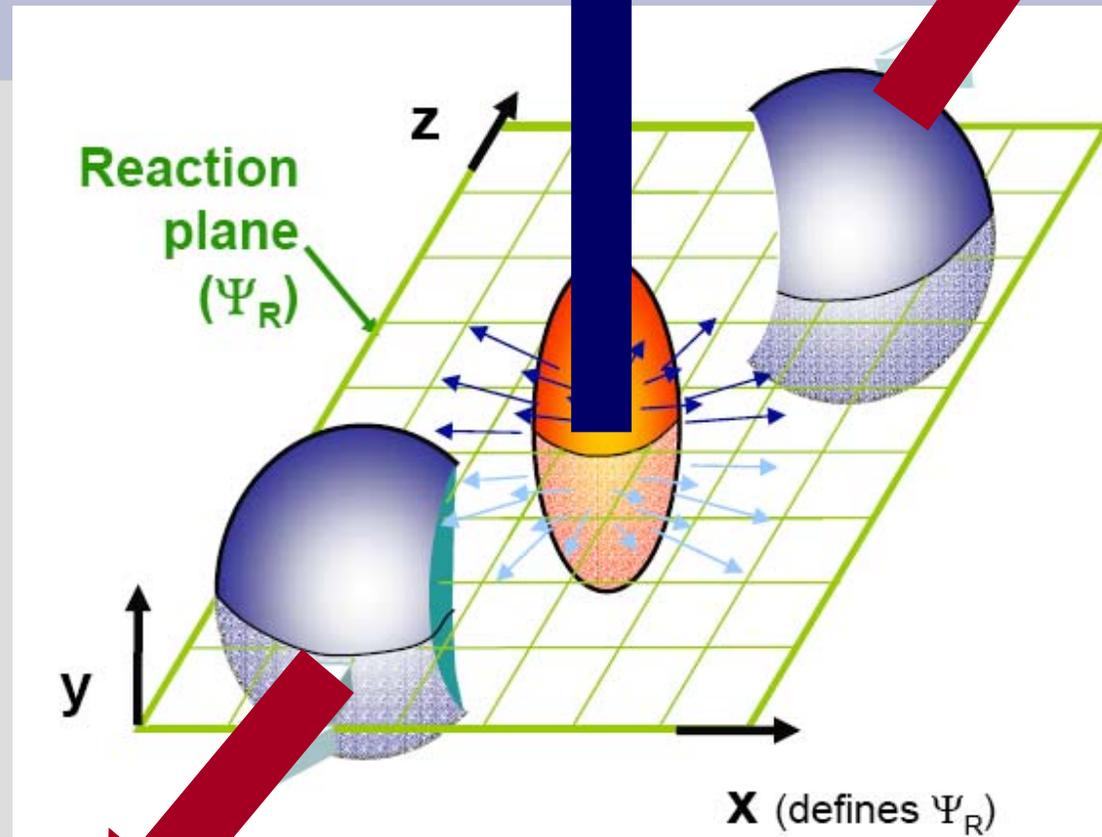
Thus we have  
two very big  
currents

**The medium is filled by electrically charged particles**

**Large orbital momentum, perpendicular to the reaction plane**

**Large magnetic field along the direction of the orbital momentum**

# Magnetic fields in non-central collisions



Two very big currents produce a very

*big magnetic field*

**The medium is filled by electrically charged particles**

**Large orbital momentum, perpendicular to the reaction plane**

**Large magnetic field along the direction of the orbital momentum**

# Magnetic forces are of the order of strong interaction forces

*first time in my life I see such effect*

$$eB \approx \Lambda_{QCD}^2$$

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We expect the influence of magnetic field on strong interaction physics

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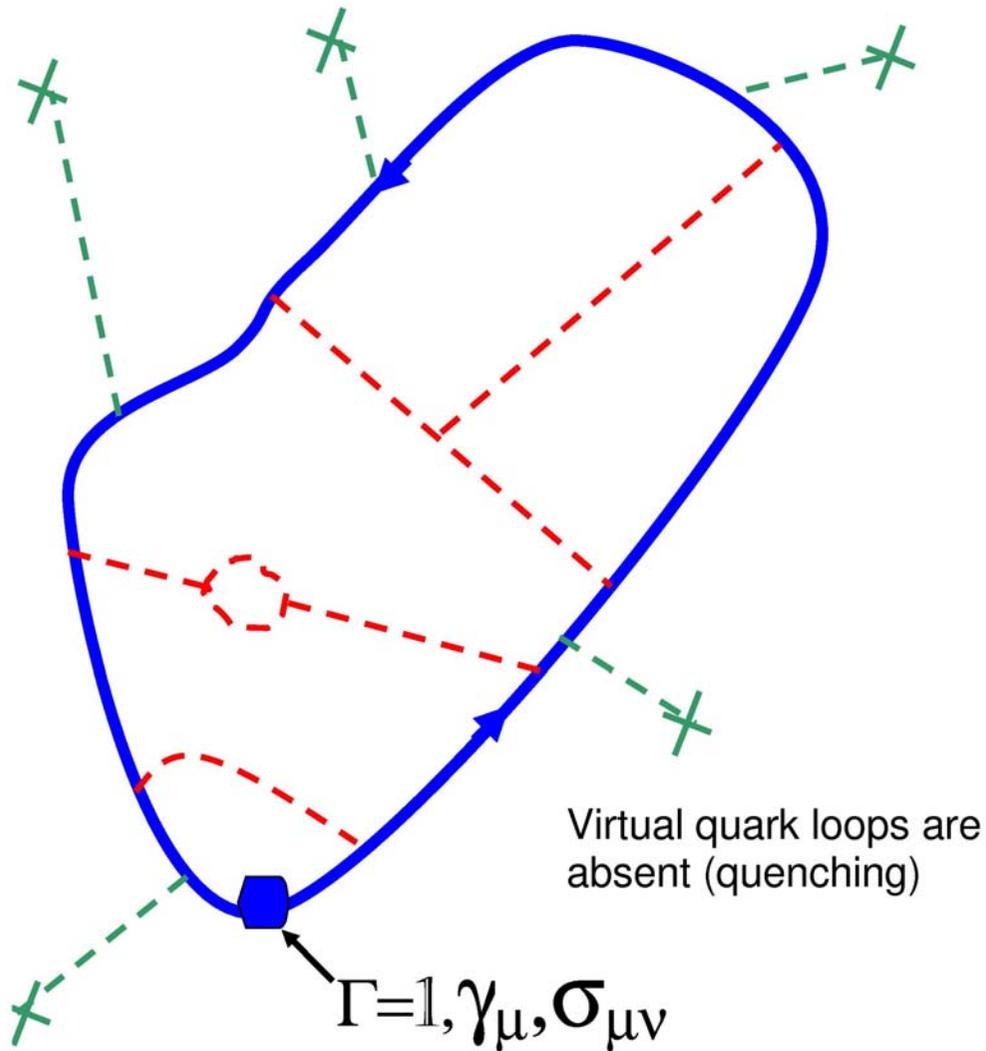
*first time in my life I see such effect*

$$eB \approx \Lambda_{QCD}^2$$

**We expect the influence of magnetic field on strong interaction physics**

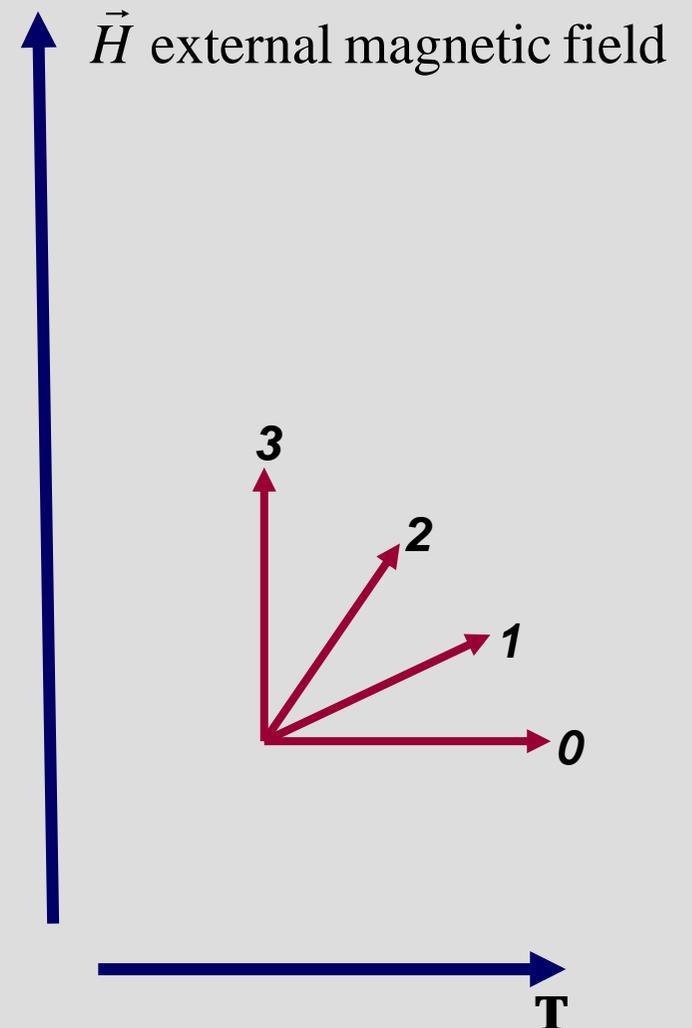
**The effects are nonperturbative,  
it is impossible to perform analytic calculations  
and we use**

**LATTICE CALCULATIONS**

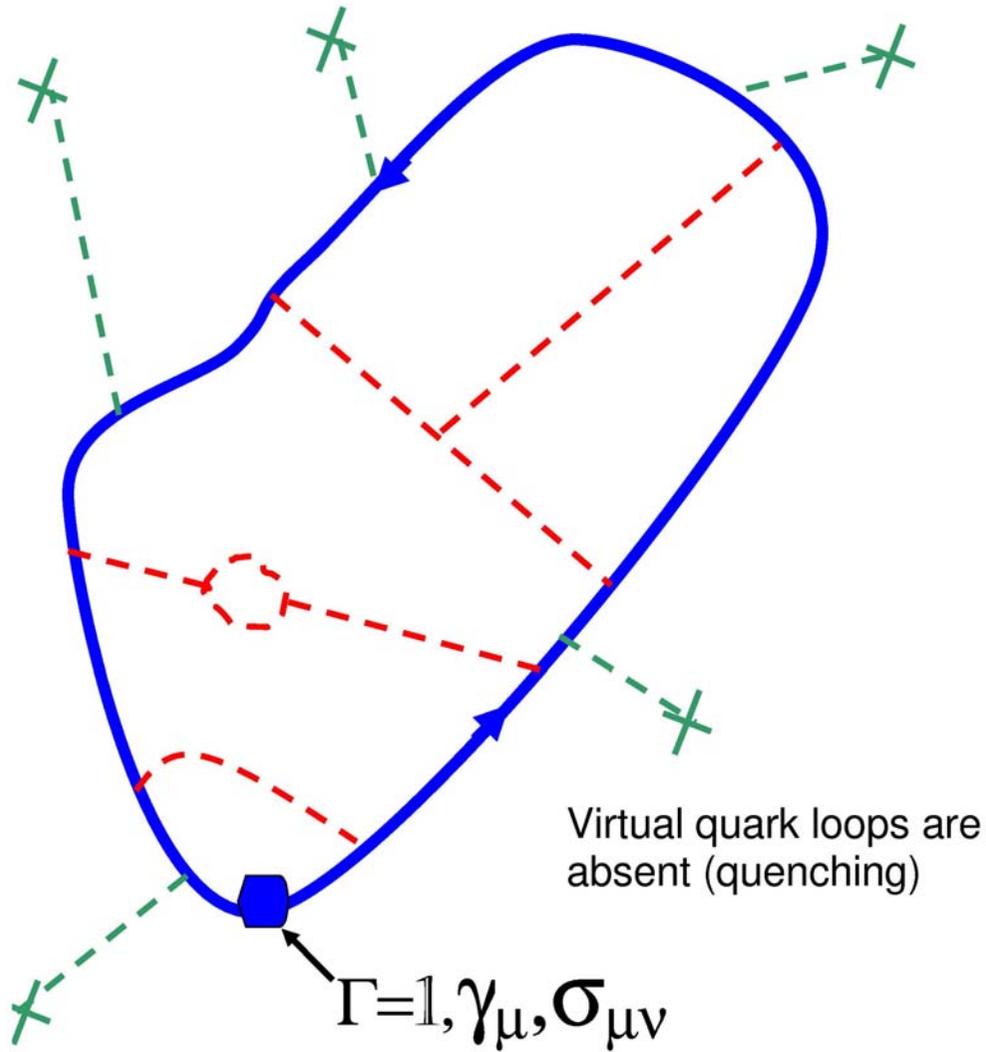


-  External quark
-  Virtual gluon
-  Photon (external magnetic field)

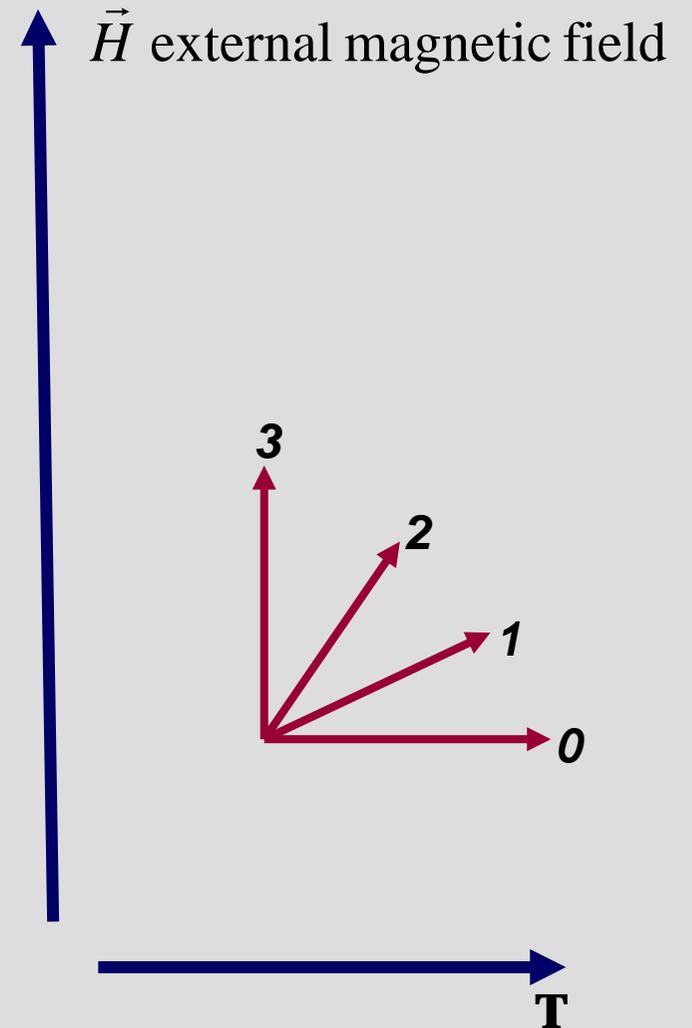
We calculate  $\langle \bar{\psi} \Gamma \psi \rangle$ ;  $\Gamma = 1, \gamma_\mu, \sigma_{\mu\nu}$   
 in the external magnetic field and in the presence of the vacuum gluon fields



We consider SU(2) gauge fields and quenching approximation



- External quark
- Virtual gluon
- Photon (external magnetic field)



# Quenched vacuum, overlap Dirac operator, external magnetic field

$$eB = \frac{2\pi qk}{L^2}; eB \geq 250 \text{ Mev}$$

# Chiral Magnetic Effect

[Fukushima, Kharzeev, Warringa, McLerran '07-'08]

**Electric current appears at regions**

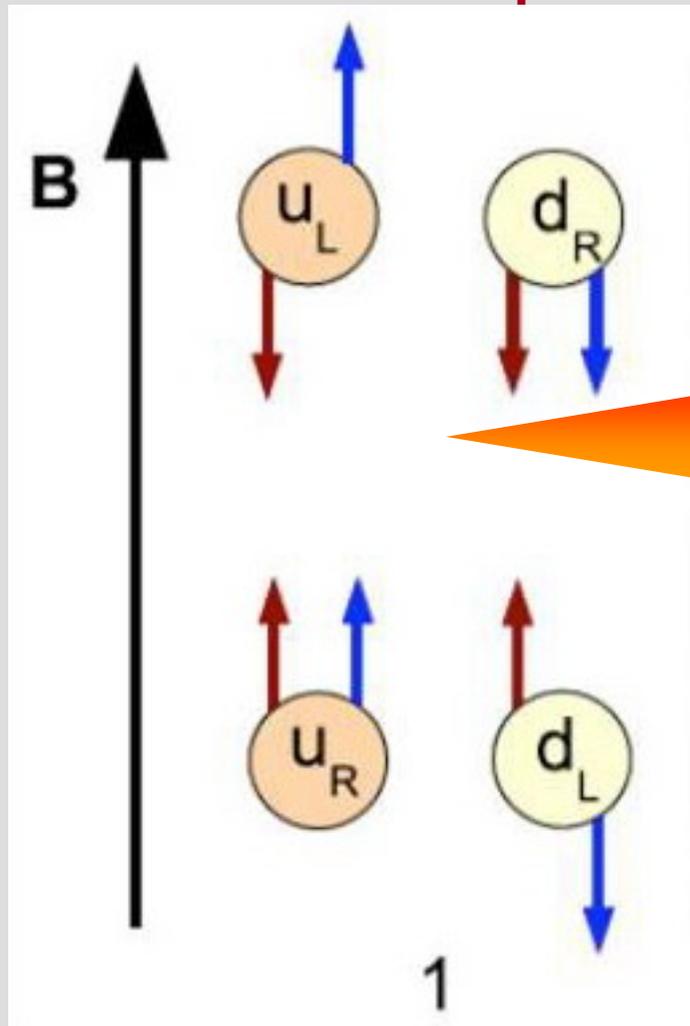
- 1. with non-zero topological charge density**
- 2. exposed to external magnetic field**

***Experimentally observed at RHIC :***

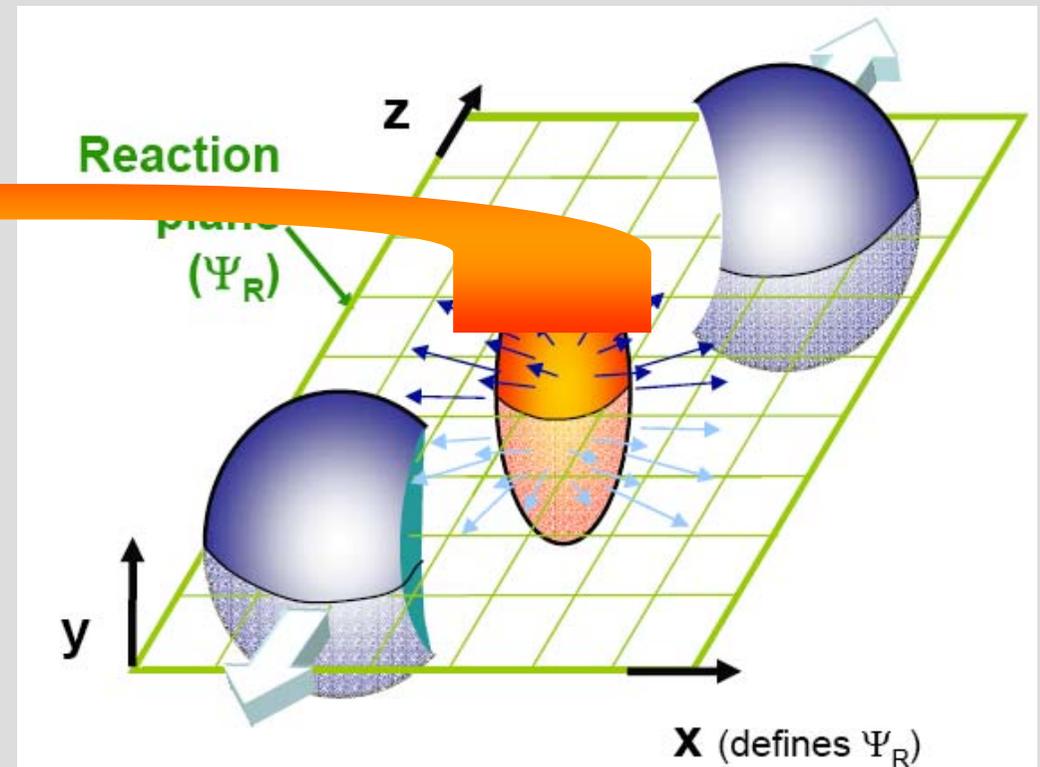
**charge asymmetry of produced particles at heavy ion collisions**

# Chiral Magnetic Effect by Fukushima, Kharzeev, Warringa, McLerran

## 1. Massless quarks in external magnetic field.

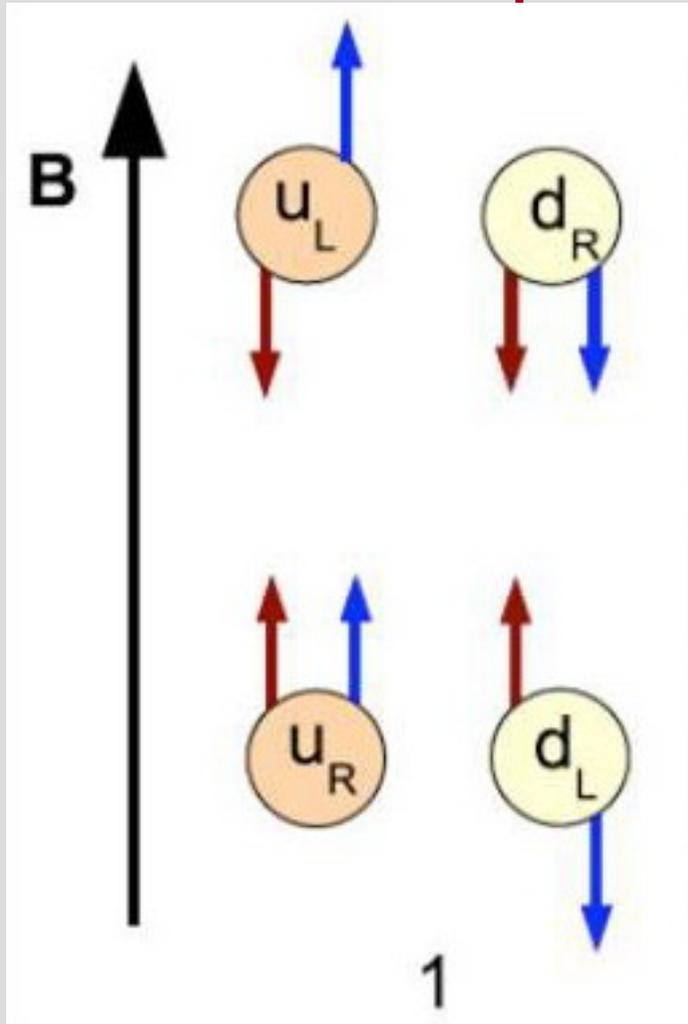


**Red: momentum**      **Blue: spin**



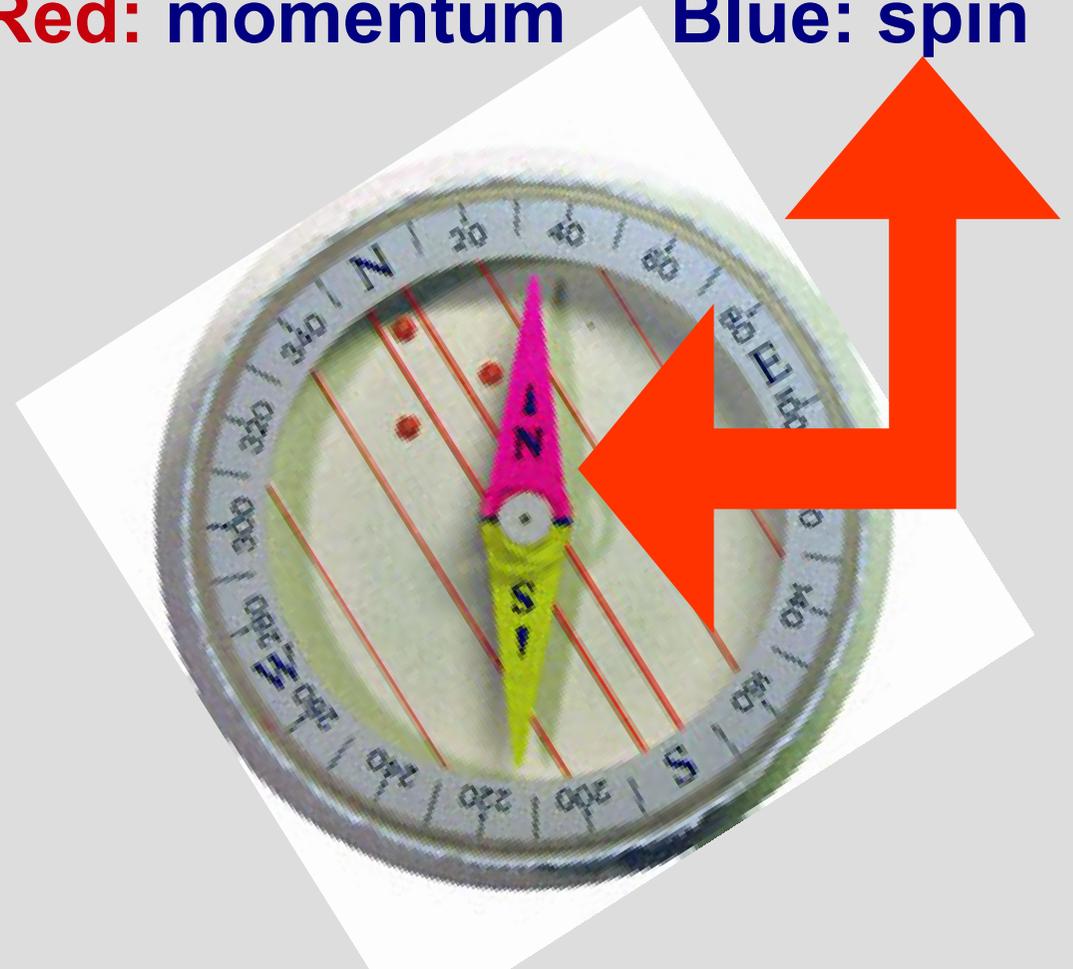
# Chiral Magnetic Effect by Fukushima, Kharzeev, Warringa, McLerran

## 1. Massless quarks in external magnetic field.



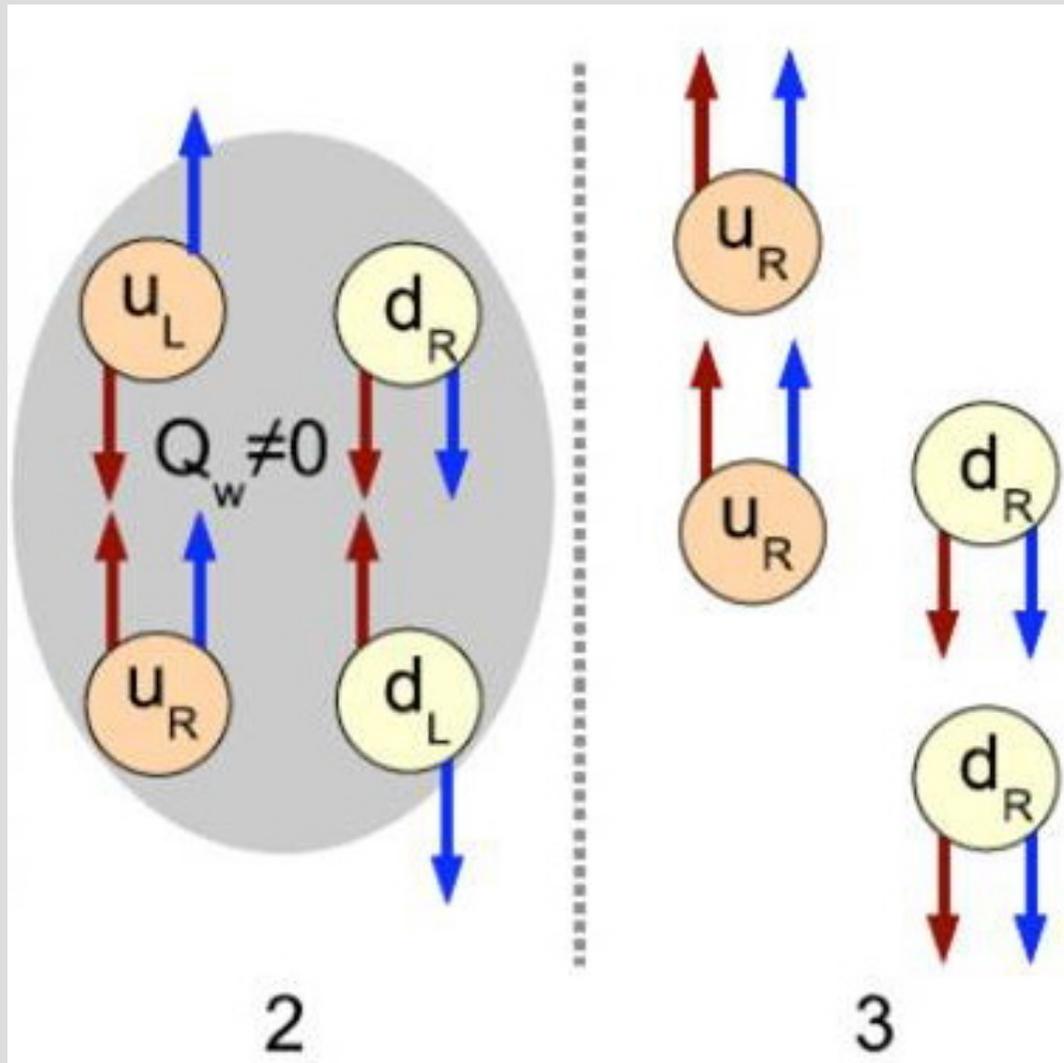
**Red: momentum**

**Blue: spin**



# Chiral Magnetic Effect by Fukushima, Kharzeev, Warringa, McLerran

## 2. Quarks in the instanton field.



**Red:** momentum  
**Blue:** spin

**Effect of topology:**

$$u_L \rightarrow u_R$$

$$d_L \rightarrow d_R$$

# Chiral Magnetic Effect by Fukushima, Kharzeev, Warringa, McLerran

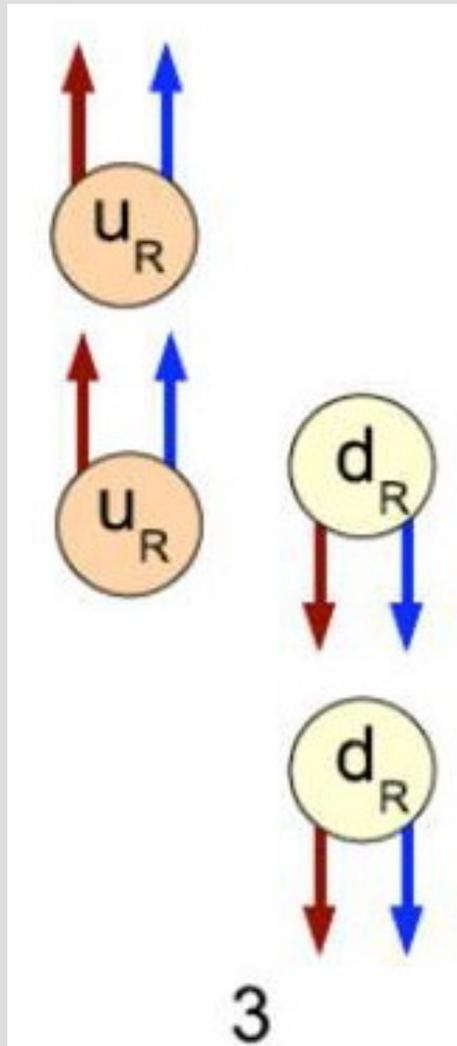
## 3. Electric current along magnetic field

**Red:** momentum  
**Blue:** spin

**Effect of topology:**

$$u_L \rightarrow u_R$$

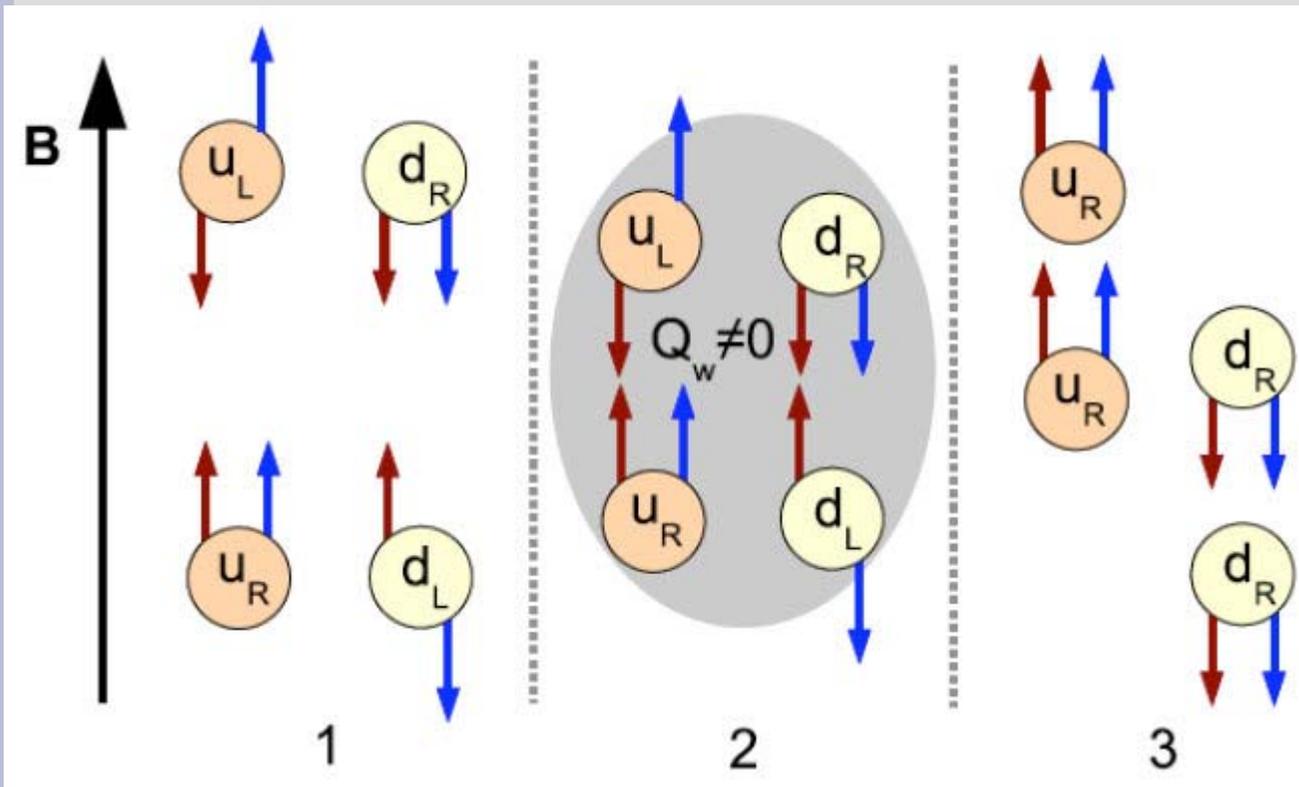
$$d_L \rightarrow d_R$$



**u-quark:  $q=+2/3$**   
**d-quark:  $q= - 1/3$**

# Chiral Magnetic Effect by Fukushima, Kharzeev, Warringa, McLerran

## 3. Electric current is along magnetic field In the *instanton* field



**Red:** momentum  
**Blue:** spin

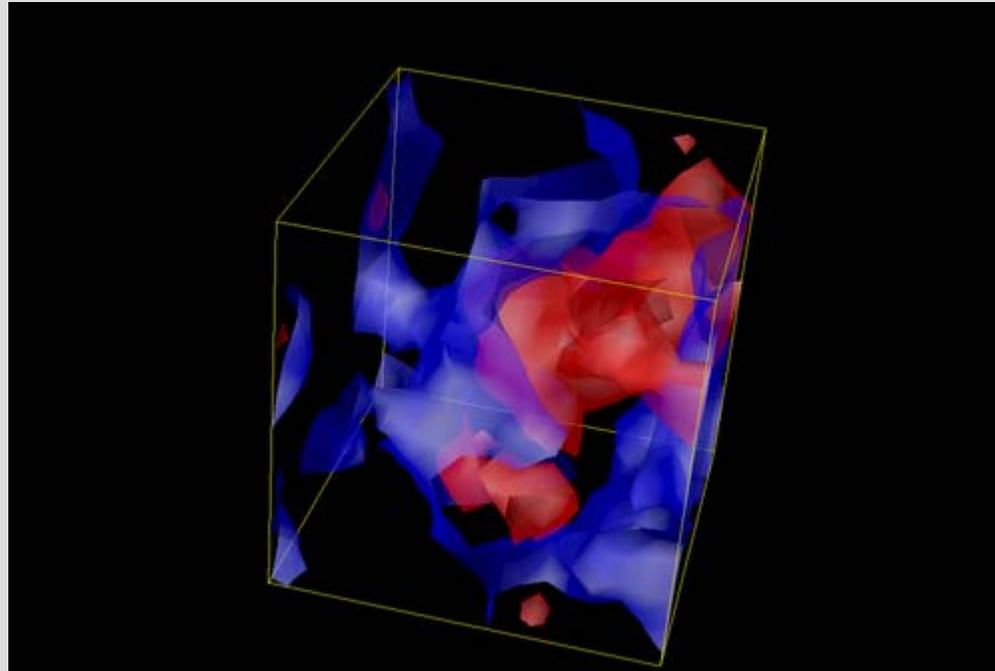
**Effect of topology:**

$$u_L \rightarrow u_R$$

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**u-quark:**  $q = +2/3$   
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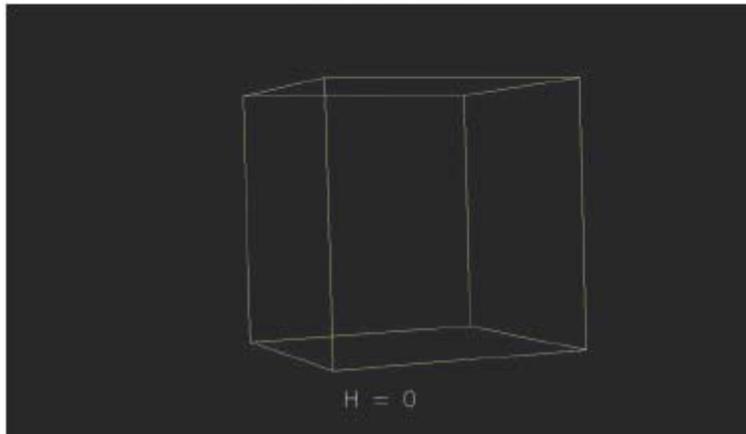
# Topological charge density in quantum QCD vacuum has fractal structure



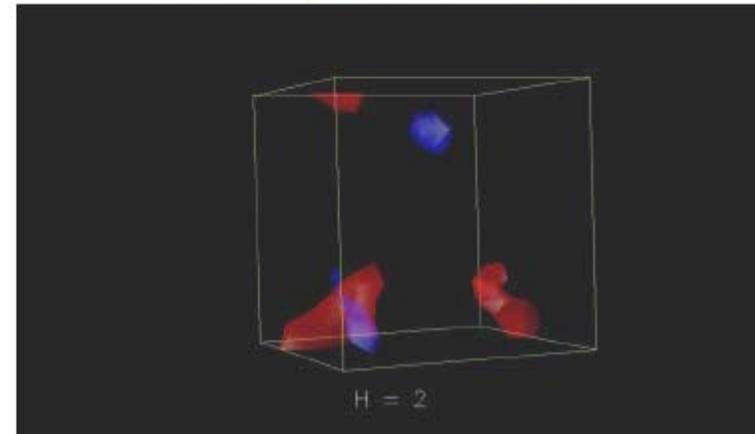
# Chiral Magnetic Effect on the lattice, charge separation

Density of the electric charge vs. magnetic field

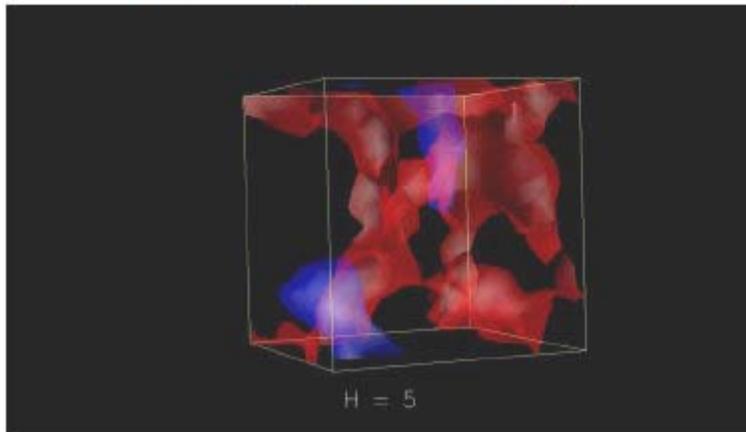
$$B = 0$$



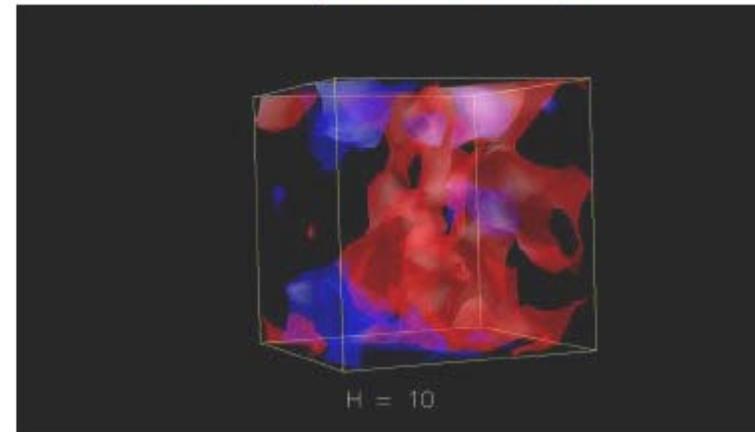
$$B = (500 \text{ MeV})^2$$



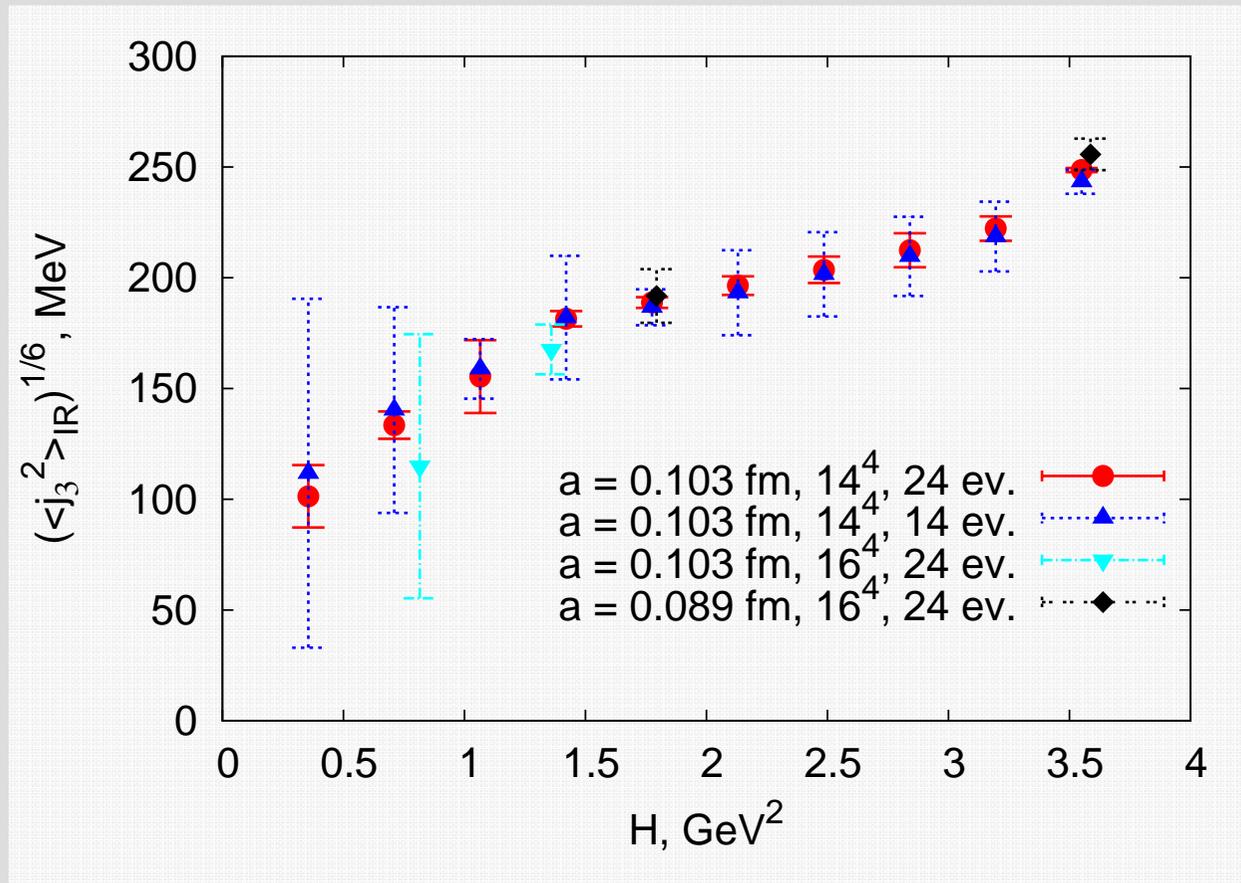
$$B = (780 \text{ MeV})^2$$



$$B = (1.1 \text{ GeV})^2$$



# Chiral Magnetic Effect on the lattice, numerical results

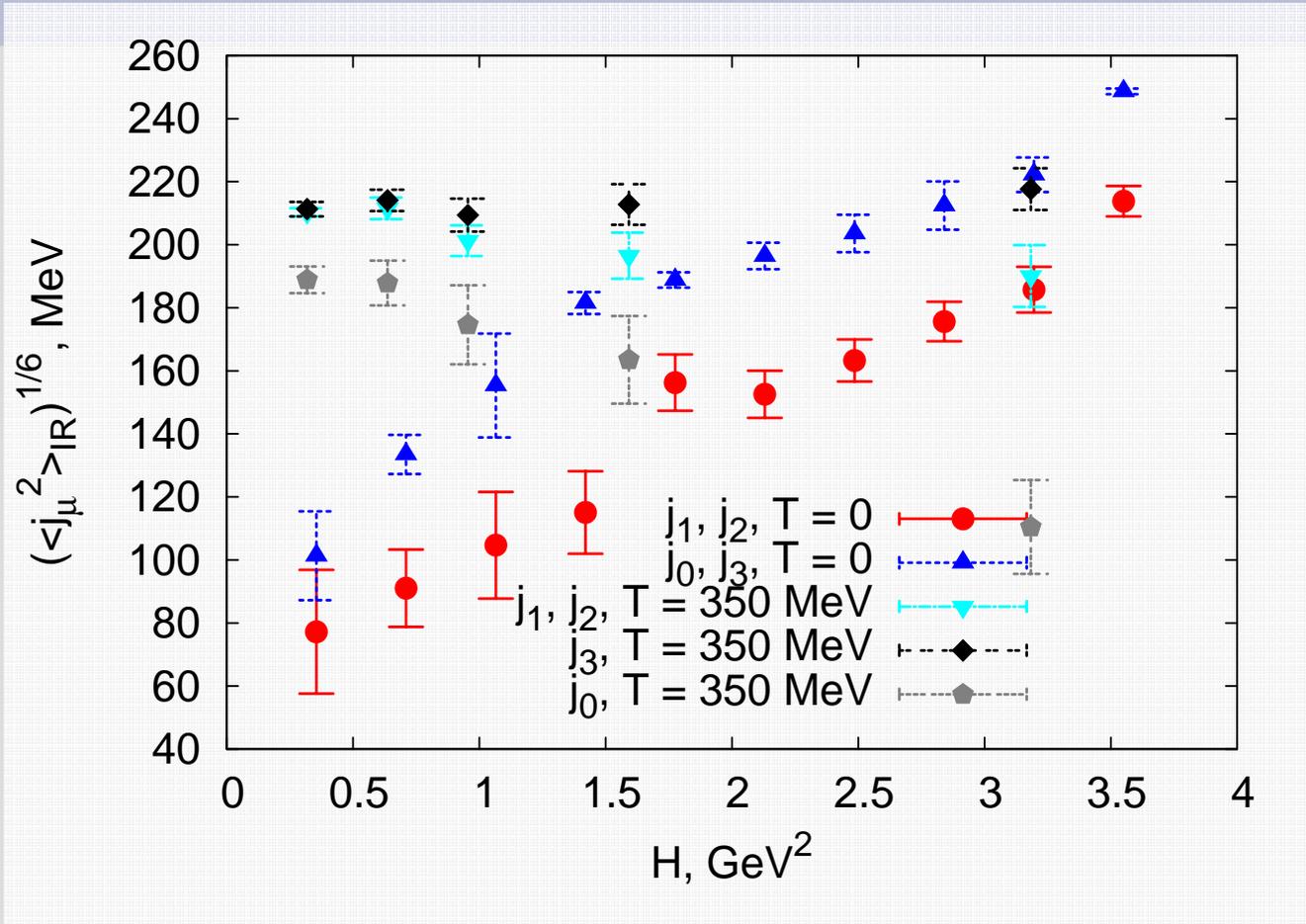


Regularized electric current:

$$\langle j_3^2 \rangle_{IR} = \langle j_3^2(H, T) \rangle - \langle j_3^2(0, 0) \rangle, \quad j_3 = \bar{\psi} \gamma_3 \psi$$

# Chiral Magnetic Effect on the lattice, numerical results

$T=0$   
 $F_{12} \neq 0$   
 $\langle j_1^2 \rangle = \langle j_2^2 \rangle$   
 $\langle j_3^2 \rangle = \langle j_0^2 \rangle$

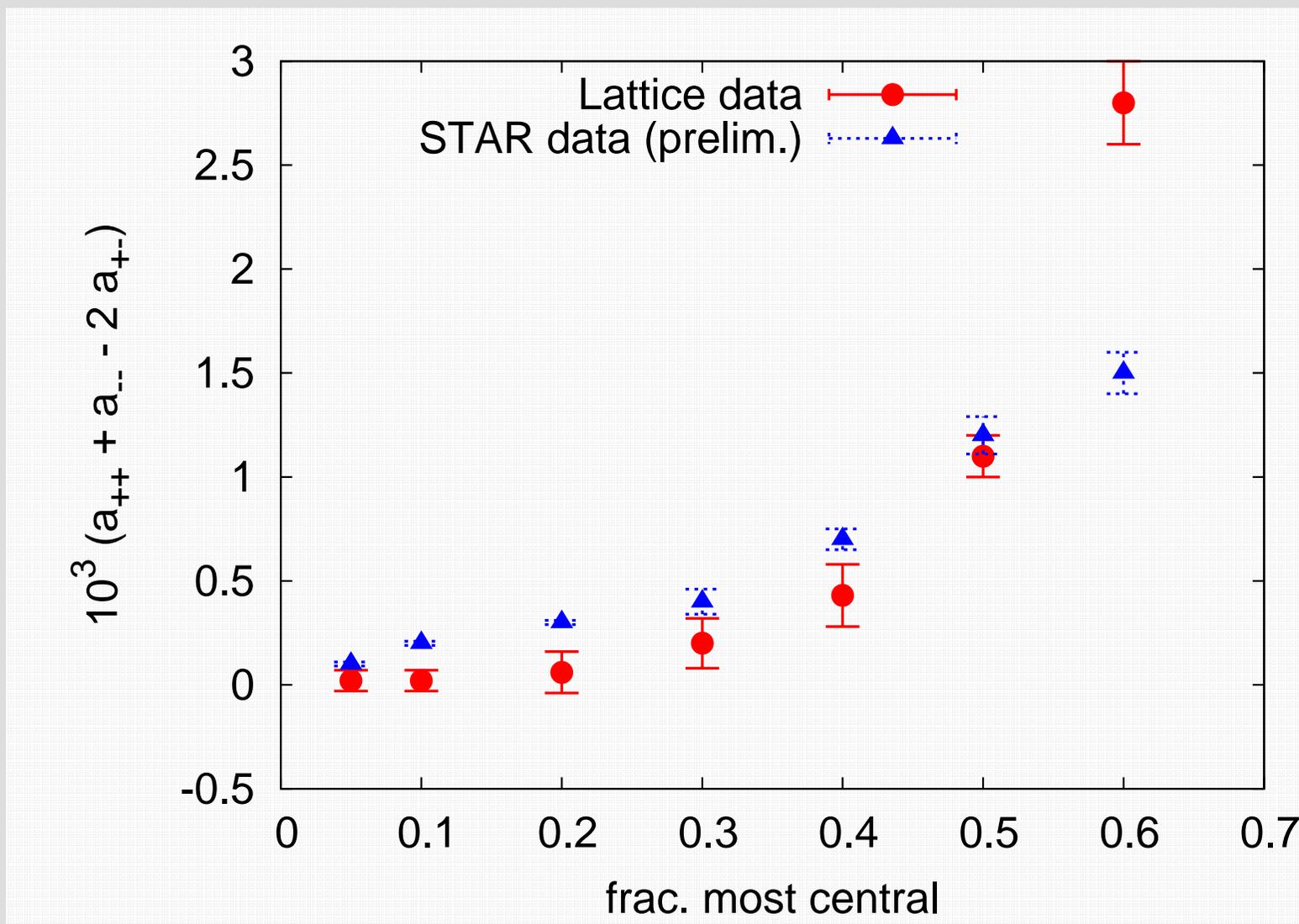


$T>0$   
 $F_{12} \neq 0$   
 $\langle j_1^2 \rangle = \langle j_2^2 \rangle$   
 $\langle j_3^2 \rangle \neq \langle j_0^2 \rangle$

Regularized electric current:

$$\langle j_i^2 \rangle_{IR} = \langle j_i^2(H, T) \rangle - \langle j_i^2(0, 0) \rangle, \quad j_i = \bar{\psi} \gamma_i \psi$$

# Chiral Magnetic Effect, EXPERIMENT VS LATTICE DATA (Au+Au)

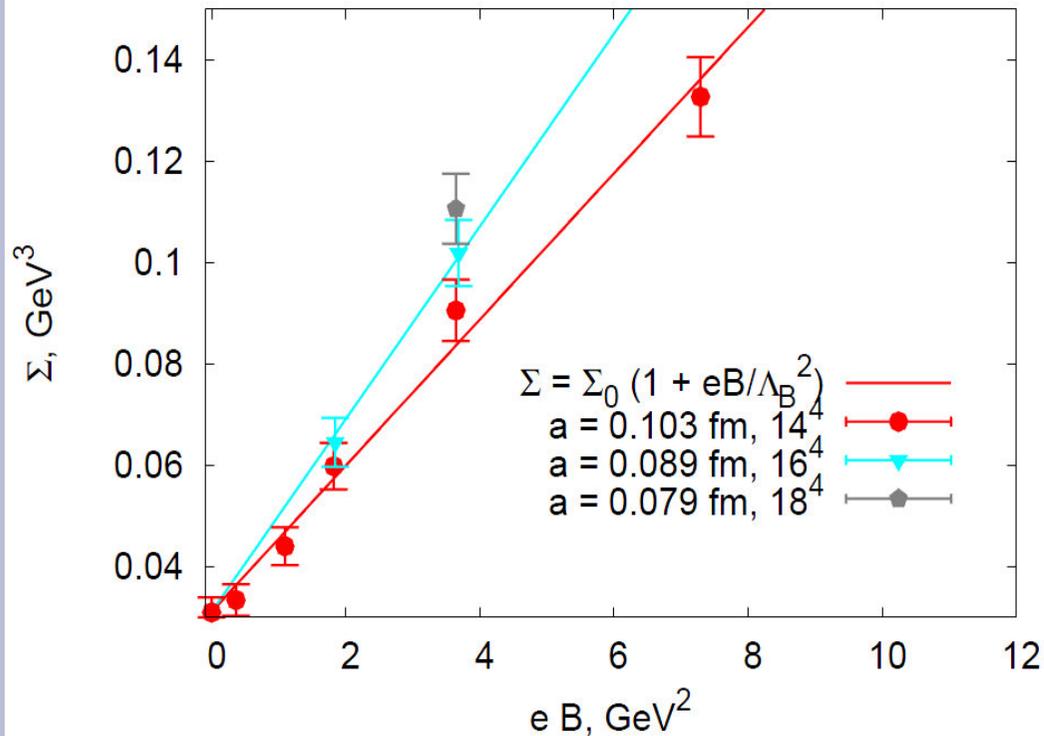


## 2. Chiral condensate in QCD

$$\Sigma = - \langle \bar{\psi} \psi \rangle$$

$$m_{\pi}^2 f_{\pi}^2 = m_q \langle \bar{\psi} \psi \rangle$$

# Chiral condensate vs. field strength



$$\Sigma = \Sigma_0 \left(1 + \frac{eB}{\Lambda_B^2}\right)$$

- Our value for  $\Lambda_B$ :

$$\Lambda_B^{\text{fit}} = (1.41 \pm 0.14 \pm 0.20) \text{ GeV}$$

- $\chi$ PT result:

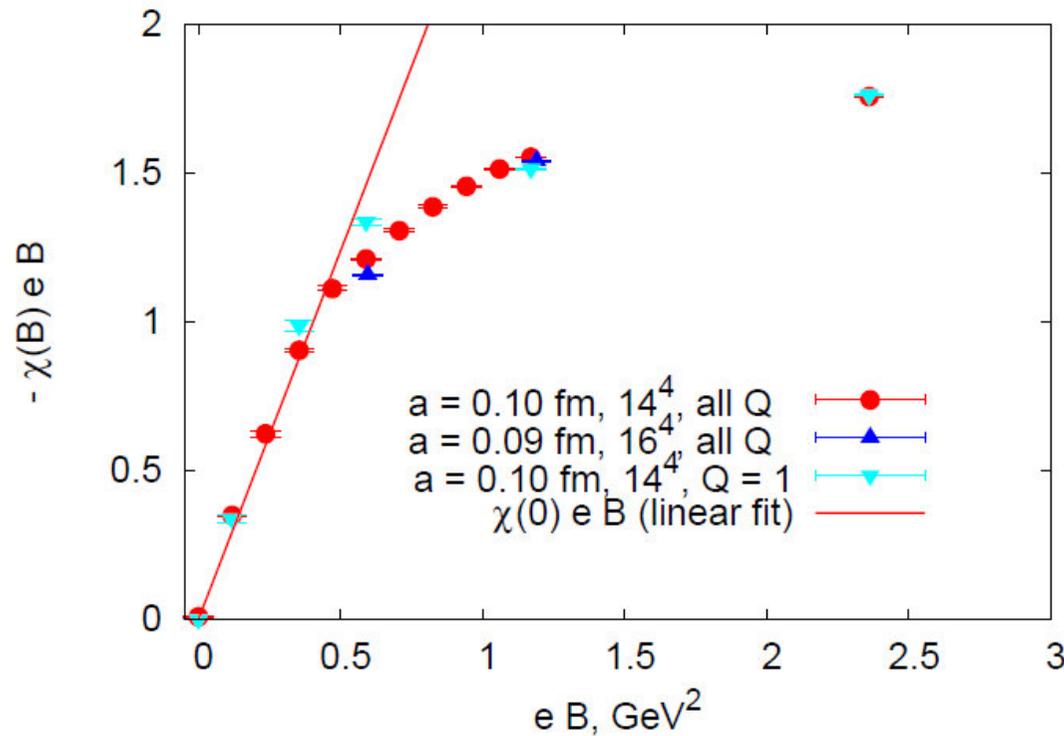
$$\Lambda_B^{\chi PT} = 1.96 \text{ GeV} \quad (F_\pi = 130 \text{ MeV} - \text{real world})$$

$$\Lambda_B^{\chi PT} = 1.36 \text{ GeV} \quad (F_\pi = 90 \text{ MeV} - \text{quenched})$$

- Chiral condensate at  $B = 0$ :  $\Sigma_0^{\text{fit}} = [(310 \pm 6) \text{ MeV}]^3$

**We are in agreement with the chiral perturbation theory: the chiral condensate is a linear function of the strength of the magnetic field!**

### 3. Magnetization of the vacuum as a function of the magnetic field



Spins of virtual quarks turn parallel to the magnetic field



$\langle \bar{\psi} \psi \rangle \chi = -46(3) \text{ Mev} \leftrightarrow$  our result

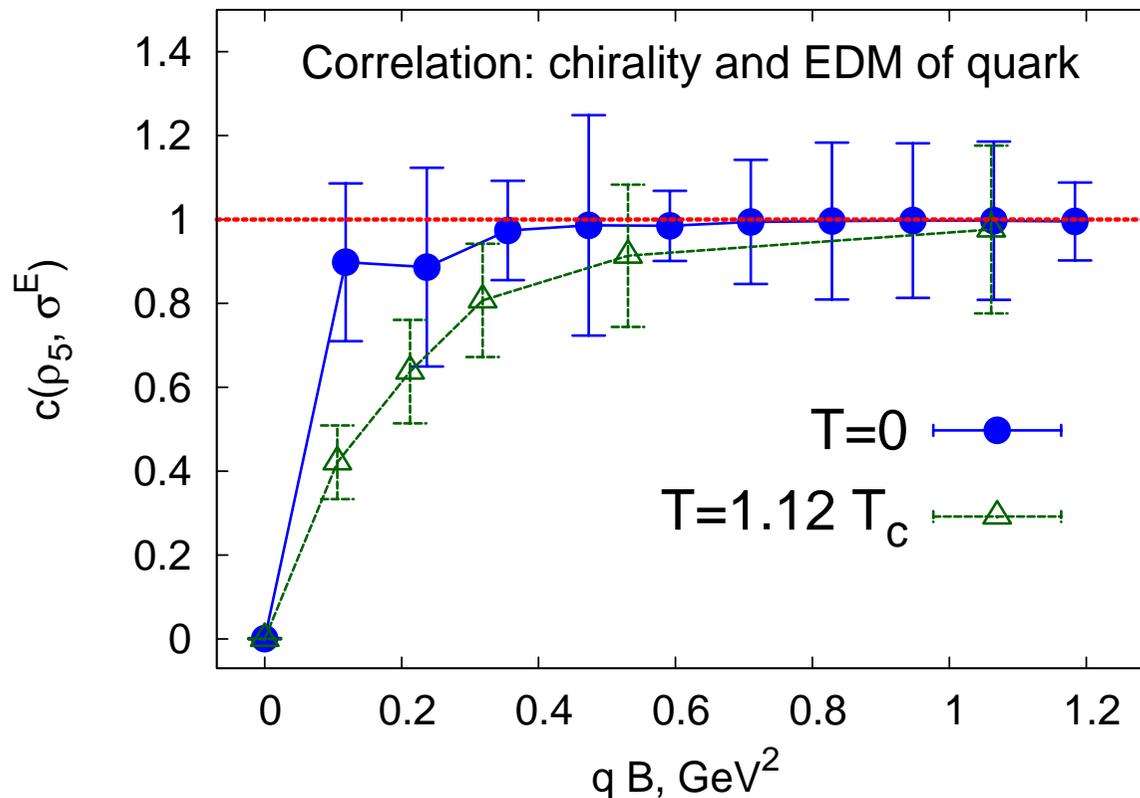
$\langle \bar{\psi} \psi \rangle \chi \approx -50 \text{ Mev} \leftrightarrow$  QCD sum rules

(I. I. Balitsky, 1985, P. Ball, 2003.)

# 4. Generation of the anomalous quark electric dipole moment along the axis of magnetic field

Large correlation between square of the electric dipole moment

$$\sigma_{0i} = i\bar{\psi}[\gamma_0, \gamma_i]\psi \quad \text{and chirality} \quad \rho_5 = \bar{\psi}\gamma_5\psi$$



# Results

1. We observe signatures of the Chiral Magnetic Effect, but the physics may differ from the model of Kharzeev, McLerran and Warringa ([arXiv:0907.0494](#), *Phys.Rev.D*79:106003,2009 )
2. We observe that the chiral condensate is proportional to the strength of the magnetic field, the coefficient of the proportionality agrees with Chiral Perturbation Theory. Microscopic mechanism for the chiral enhancement is the localization of fermion modes in the vacuum ([arXiv:0812.1740](#), *Phys.Lett. B*)
3. The calculated vacuum magnetization is in a qualitative agreement with model calculations ([arXiv:0906.0488](#), *Nucl.Phys. B* 826 (2010) 313)
4. We observe very large correlation between electric dipole moment of quark and chirality ([arXiv:0909.2350](#) )

**[arXiv:0909.1808](#)**

# Conclusions

## 1. In QCD with dynamical quarks we

- a) Observe confining string breaking
- b) Find the structure of flux tubes inside baryon
- c) Calculate the critical temperature

## 2. In QCD with external magnetic field we

- a) Observe charge separation, but the physics may differ from the model of Kharzeev, McLerran and Warringa
- b) The charge separation is weaker for heavy quarks, thus it is interesting to measure in experiments charge asymmetry for S and C quarks
- c) The strong magnetic field exists only very short time  $\lesssim 1 \text{ fm}$  still it can influence physics of strong interactions