Cosmological models with gauge fields

Gauge fields. Yesterday, Today, Tomorrow. Dedicated to 70-th anniversary of A.A. Slavnov.

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Vector fields in cosmology

- The idea of cosmological phase transitions was invoked to explain inflationary stage of the early universe which provides solution of the horizon problem. The simplest way to implement this idea in Friedman cosmology within the Gauge theory framework is to introduce the homogeneous and isotropic condensate of the Higgs field with a potential. It turned out that the potential has to be strongly fine-tuned to fit cosmological parameters, and moreover, a single scalar field is not sufficient for this purpose.
- Modern theories provide various candidates for the inflaton such as moduli fields arising in compactification of supergravities/string theory, apart from the scalar fields of standard model and its supersymmetric generalizations. Still physical nature of the inflaton is far from being uniquely understood, and motivation for a choice of the inflaton potential is mostly only qualitative. Similar problems arise in attempts to understand dark energy.
- Meanwhile, homogeneous and isotropic vector fields can be invoked in cosmology as well. Massless vector fields are constituents of the Standard model, and a variety of vector fields is suggested by supergravity/strings. U(1) (Maxwell) field $F_{\mu\nu}$ with a potential $V(A_\mu^2)$ may generate anisotropic inflation for some tuned choice of V (Ford, 89'). However, this suggestion has two drawbacks: first, the potential violates gauge invariance, second, an homogeneous vector field has anisotropic stresses, so to embed the system into isotropic Friedman cosmology one has to introduce at least a triplet of vector fields and average over directions.

Conformal (non) invariance

- Another problem with vector fields is conformal invariance which holds for Maxwell lagrangian in four dimensions. Conformal matter leads to hot Friedman cosmology exactly as the photon gas, so to get an accelerated expansion one has to violate conformal invariance. Actually, the above potential term does this.
- Recently several attempts were made to promote vector fields with phenomenologically violated conformal invariance as candidate for dark energy: Novello, Bergliaffa and Salim 03', Elizalde, Odintsov, Lidsey and Nojiri 03', Kiselev 04', Fuzfa and Alimi 06', Zhao and Zhang 06', H. Wei and R. G. Cai 06', Jimenez and Maroto 08' etc.
- More natural mechanisms of breaking of conformal invariance include quantum corrections in gauge theories or Born-Infeld actions for gauge fields on the branes in string theory. Such models typically interpolate betwen string equation of state $P = -\epsilon/3$ and hot equation of state $P = \epsilon/3$
- Our novel model here is Weinberg-Salam cosmology with the coupled SU(2) YM and complex doublet Higgs, which combines features of both the scalar inflation and the YM dynamics leading to interesting regimes such as transient periods of accelerated expansion and cyclic evolution with chaotically distributed parameters, resembling the multiverse developed in time.

SU(2) homogeneous and isotropic field

A very natural vector triplet is provided by the SU(2) Yang-Mills. Unlike the U(1) case, classical SU(2) vector field configuration exists which is compatible with space homogeneity and isotropy of the FRW metric

$$ds^2 = a^2(\eta)\left(d\eta^2 - dl_k^2\right)$$

(where $k = -1,\ 0,\ 1$ stand for open, flat and closed spatial geometry) without averaging. This was shown for k=1,0 by Cervero and Jacobs 78', Henneaux 82', Hosotani 84'; and for all k by Gal'tsov and Volkov 91' (some later work on the EYM FRW cosmology: Bertolami, Mourao, Moniz, Cavaglia, de Alfaro, Filippov, Kapetanakis, Koutsoumbas, Lukas, Mayr, Ding). The configuration is parametrized by a single scalar function $h(t)$ and contains both color electric and magnetic components

$$E_i^a = \dot{h}\,\delta_i^a, \quad B_i^a = (k - h^2)\,\delta_i^a.$$

The reduced YM lagrangian in the conformal frame reads

$$L = \frac{3}{8\pi a^4}\left(\dot{h}^2 - (k - h^2)^2\right)$$

so the electric part corresponds to the kinetic term in the reduced action, while the magnetic part — to the potential term. It is worth noting that this configuration is unique up to scaling.

Cold matter for hot Universe (Gal'tsov and Volkov 91')

The standard YM action is conformally invariant, so the above configuration in the context of FRW cosmology gives rise to the EOS of the photon gas

$$P = \epsilon/3 \quad \text{with} \quad \epsilon = \frac{3}{8\pi a^4}\left[\dot{h}^2 + (k - h^2)^2\right]$$

Thus one obtains the hot Universe driven by cold matter.
The YM equations reduce to EOM of a particle in the potential well

$$V = (k - h^2)^2$$

which in the closed case is a double well potential. Two minima correspond to neighboring topologically different vacua. Depending on the "particle" total energy, it (classically) oscillates either around a single vacuum, or around both of them. In the flat and open cases the potential has one minimum at $h = 0$ with $V = 0$ for flat and $V = 1$ for open. The Chern-Simons 3-form

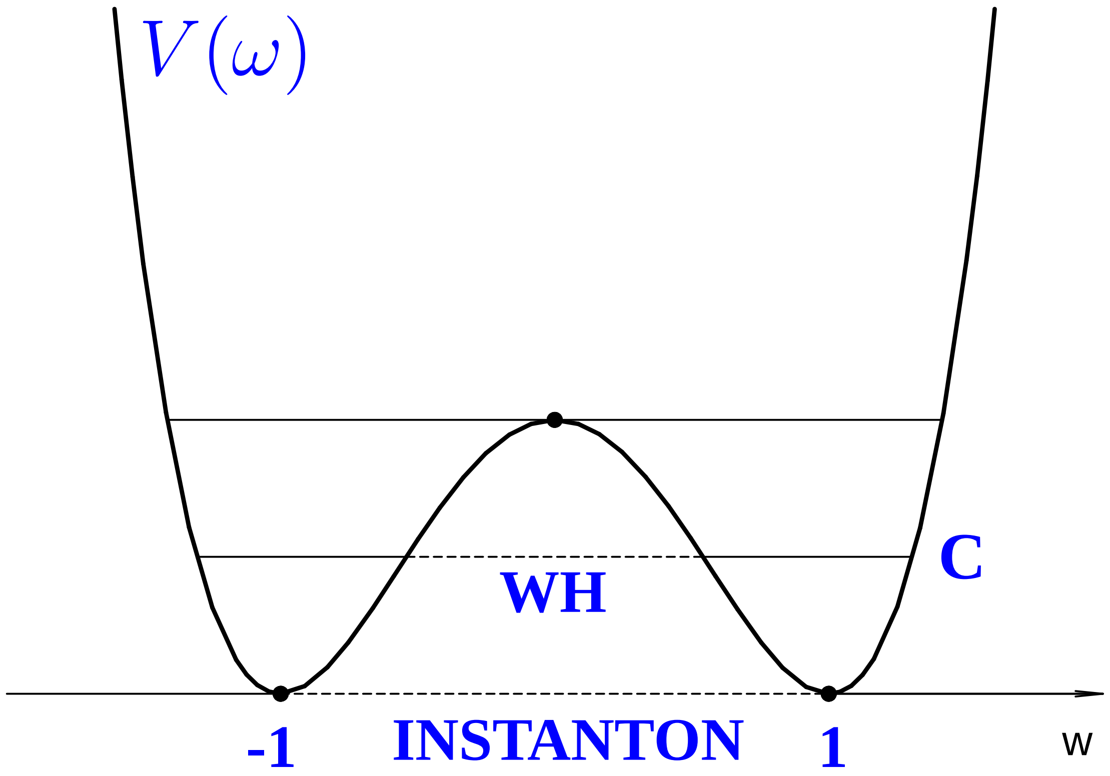
$$\omega_3 = \frac{e^2}{8\pi^2}\mathrm{Tr}\left(A \wedge dA - \frac{2ie}{3}A \wedge A \wedge A\right),,$$

satisfying $d\omega_3 = \frac{e^2}{8\pi^2}\mathrm{Tr}F \wedge F$, is non-trivial in the case $k = 1$ (winding number of the map $SU(2) \to S^3$):

$$N_{CS} = \int_{S^3} \omega_3 = \frac{1}{4}(h+1)^2(2-h)$$

Vacuum $h = -1$ is topologically trivial: $N_{CS} = 0$, vacuum $h = 1$ is non-trivial: $N_{CS} = 1$

Closed FRW Universe



EYM sphalerons

- Self-gravitating YM field gives rise to particle-like solutions analogous to sphalerons in the Weinberg-Salam theory (Gal'tsov and Volkov 91'). The role of Higgs is played by gravity. Creation and decay of these sphalerons generates a transition of the YM field from one vacuum to another and is accompanied by the fermion number non-conservation
- In the EYM cosmology an unstable state $w = 0$ (w-particle sitting at the top of the barrier between two vacua) is cosmological sphaleron (Gibbons and Steif 93'). It has topological charge $N_{CS} = 1/2$ like sphalerons in the Weinberg-Salam theory. The EOM
$$\ddot{h} = 2h(1 - h^2)$$
is solved by $h = 0$, this correspond to the h-particle total energy
$$\dot{h}^2 + (1 - h^2)^2 = 1$$
The YM field in this case is purely magnetic. A more general solution with the same energy describes rolling down of w-particle (sphaleron decay): $h = \sqrt{2}/\cosh(\sqrt{2}\eta)$, where η is conformal time. Rolling down to vacuum $h = 0$ takes an infinite time, while the corresponding full cosmological evolution $a = \sqrt{\frac{4\pi g}{g^2}} \sin\eta$ takes finite time. Thus cosmological sphaleron is quasi stable. This conclusion is not modified if a positive cosmological constant is added (Ding 94')

Instanton and wormholes Donets and Gal'tsov 92'

- Instanton corresponds to tunneling between the neighboring vacua $h = \pm 1$. It is a self-dual Euclidean YM configuration for which the stress tensor is zero, therefore conformally flat gravitational field can be added just as a background. Tunneling solutions at higher excitation levels $0 < C \leq 1$ are not self-dual. In flat space-time they are known as meron ($C = 1,\ N_{CS} = 1/2$), which is the Euclidean counterpart of the cosmological sphaleron, and the so-called nested merons $0 < C < 1,\ 1/2 < N_{CS} < 1.$
- The energy-momentum tensor of the meron is non-zero, and in flat space this solution is singular. When gravity is added, the singularity at the location of a meron expands to a wormhole throat, and consequently, the Euclidean topology of the space-time transforms to that of a wormhole. Topological charge of the meron wormholes is zero, the charge of the meron being swallowed by the wormhole (Hosoya and Ogura 89', Verbin and Davidson 90', Gupta et al 90', Rey 90').
- The total action for these wormholes is divergent because of slow fall-off of the meron field at infinity, so the amplitude of creation of a baby universe associated with the Euclidean wormholes is zero. However, when a positive cosmological constant is added (inflation) the action becomes finite due to compactness of the space. Such solutions can be interpreted as describing tunneling between the de Sitter space and the hot FRW universe.

The wormhole action

- In the Euclidean regime the EOM-s for ΛEYM theory are separable and admit the integrals (in conformal time)

$$\dot{h}^2 - (h^2-1)^2 = -C, \quad \dot{a}^2 + (\Lambda a^4/3 - a^2) = -C/(e^2 m_{Pl}^2)$$

  Solutions describe independent tunneling for the YM variable h, and the cosmological radius a with different periods $T_h,\ T_a$ depending on the excitation level C. To be wormholes, they must obey a quantization condition $n_h T_h = n_a T_a$ with two integers (Verbin and Davidson 89')

- For a specific value of the cosmological constant

$$\Lambda = \frac{3}{4} m_{Pl}^2 e^2$$

  it was found that $T_a = T_h$ for all $C \in [0,1]$ (Donets, Gal'tsov 92'). In particular, for $C = 1$ (meron limit) the radius a becomes constant (Euclidean static Einstein Universe). For $C \neq 1, 0$ the solutions describe creation of baby universes.

- Remarkably, under above conditions, the total action (gravitational plus YM) is precisely zero

$$S_{YM} + S_{gr} = 0$$

  Thus, the pinching off of baby universes occurs with probability one.

Non-Abelian Born-Infeld

Open string theory suggests the replacement of the Maxwell Lagrangian density $L = -\frac{\sqrt{-g}F_{\mu\nu}F^{\mu\nu}}{16\pi}$ by the Born-Infeld one

$$L = \frac{\beta^2}{4\pi}\left(\sqrt{-\det(g_{\mu\nu} + F_{\mu\nu}/\beta)} - \sqrt{-g}\right),$$

β being the critical BI field strength ($\beta = 1/2\pi\alpha'$ in string theory). In four dimensions this is equivalent to

$$L = \frac{\beta^2}{4\pi}(\mathcal{R} - 1), \quad \mathcal{R} = \sqrt{1 + \frac{F_{\mu\nu}F^{\mu\nu}}{\beta^2} - \frac{(\tilde{F}_{\mu\nu}F^{\mu\nu})^2}{16\beta^4}}$$

In the non-Abelian case $F^{\mu\nu}$ is matrix valued, and the prescription is more complicated (symmetrized trace)

$$L_{STr} = \frac{\beta^2}{4\pi}\mathrm{STr}\left(\sqrt{-\det(g_{\mu\nu} + F_{\mu\nu}/\beta)} - \sqrt{-g}\right)$$

But qualitatively similar results are obtained with an ordinary trace prescription equivalent to summation over color indices in the field invariants $F^a_{\mu\nu}F^{a\mu\nu}$, $\tilde{F}^a_{\mu\nu}F^{a\mu\nu}$. In FRW cosmology $ds^2 = N^2dt^2 - a^2dl_3^2$ one finds (Gal'tsov, Dyadichev 03')

$$L_{Str} = -Na^3\frac{1 - 2K^2 + 2V^2 - 3V^2K^2}{\sqrt{1 - K^2 + V^2 - K^2V^2}}, \quad L = -Na^3\sqrt{1 - 3K^2 + 3V^2 - 9K^2V^2}$$

Here

$$K^2 = \frac{\dot{h}^2}{\beta^2a^2N^2}, \qquad V^2 = \frac{(h^2 - k)^2}{\beta^2a^4}$$

NBI cosmology

Homogeneous and isotropic NBI cosmology with an ordinary trace lagrangian turns out to be completely solvable by separation of variables (Dyadichev, Gal'tsov, Zorin, Zotov,02'). It lieads to an interesting EOS:

$$P = \frac{\epsilon}{3}\frac{\epsilon_c - \epsilon}{\epsilon_c + \epsilon},$$

where $\epsilon_c = \beta/4\pi$ is the critical energy density, corresponding to vanishing pressure. For larger energies the pressure becomes negative, its limiting value being

$$P = -\epsilon/3$$

(EOS of the ensemble of non-interacting isotropically distributed straight Nambu-Goto strings (indicating on the stringy origin of the NBI lagrangian). In the low-energy limit the YM EOS:

$$P = \epsilon/3$$

is recovered. Thus, the NBI FRW cosmology smoothly interpolates between the string gas cosmology and the hot Universe. The energy density is

$$\epsilon = \epsilon_c \left(\sqrt{\frac{a^4 + 3(h^2 - k)^2}{a^4 - 3\dot{h}^4}} - 1 \right),$$

From the YM (NBI) equation one obtains the following evolution equation for the energy density:

$$\dot{\varepsilon} = -2\frac{\dot{a}}{a}\,\frac{\varepsilon\,(\varepsilon + 2\varepsilon_c)}{\varepsilon + \varepsilon_c}, \quad (1)$$

which can be integrated to give

$$a^4(\varepsilon + 2\varepsilon_c)\varepsilon = \mathrm{const.} \quad (2)$$

From this relation one can see that the behavior of the NBI field interpolates between two patterns: 1) for large energy densities ($\varepsilon \gg \varepsilon_c$) the energy density scales as $\varepsilon \sim a^{-2}$; 2) for small densities $\varepsilon \ll \varepsilon_c$ one has a radiation law $\varepsilon \sim \; a^{-4}$.
Remarkably, the equation for the scale factor a can be decoupled ($g = \beta G$):

$$\ddot{a} = -\frac{2ga(\dot{a}^2 + k)}{2ga^2 + 3(\dot{a}^2 + k)}$$

and admits the first integral

$$3\left(\dot{a}^2 + k\right)^2 + 4ga^2\left(\dot{a}^2 + k\right) = C,$$

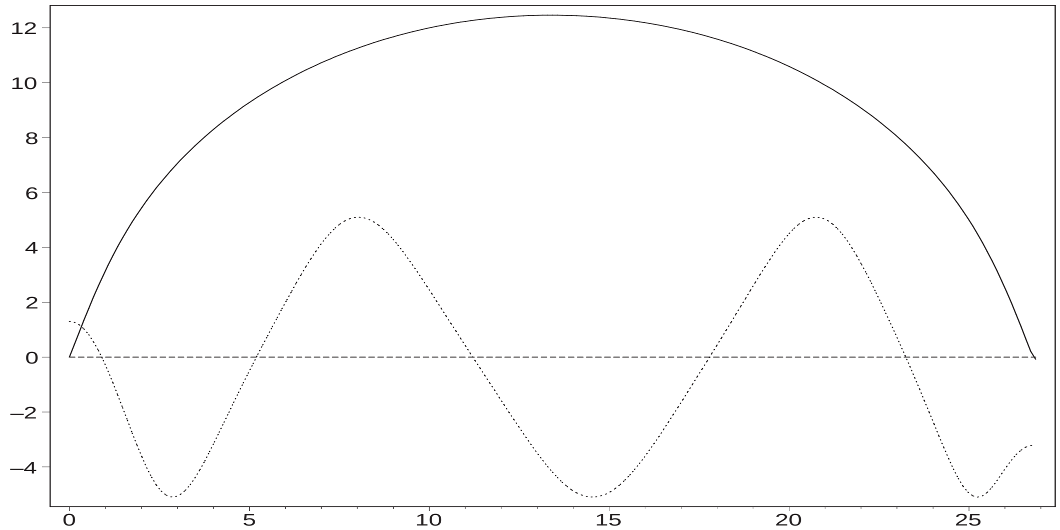
which allows to draw phase portraits for different k

Dynamics of YM field

The NBI equation for h can be integrated once, giving (in conformal time)

$$\frac{\dot{h}^2}{a_0^4 - (k - h^2)^2} = \frac{a^4}{a^4 + 3a_0^4}$$

which is then solved in terms of Jacobi elliptic functions:



The main difference with the ordinary EYM cosmology relates to small values of a. One can see that near the singularity ($a \to 0$) the YM oscillations in the NBI case slow down, while in the ordinary YM cosmology the frequency remains constant in the conformal gauge. Near the singularity $h = h_0 + \frac{b_0 \alpha}{6(k+b_0^2)} t^2 + +O(t^4)$ where $\alpha = \pm\sqrt{3(k + b_0^2)^2 - 4g^2(k - h_0^2)^2}$, h_0 is a free parameter, b_0 is a metric expansion parameter.

NBI on the brane

Replacement of the standard YM lagrangian by the Born-Infeld one breaks conformal symmetry, providing deviation from the hot EOS and creating negative pressure. Surprisingly enough, putting the same NBI theory into the RS2 framework gives rise to an exact restoration of the conformal symmetry by the brane non-linear corrections (Gal'tsov and Dyadichev 03'). The action is

$$S = \lambda \widetilde{\mathrm{Tr}} \int \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu}/\beta)}\, d^4x - \kappa^2 \int (R_5 + 2\Lambda_5)\sqrt{-g_5},\, d^5x$$

where the brane tension λ plays a role of the BI critical energy density. The constraint equation reads

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\kappa^2}{6}\Lambda + \frac{\kappa^4}{36}(\lambda+\varepsilon)^2 + \frac{\mathcal{E}}{a^4} - \frac{k}{a^2},$$

where $\mathcal{E}$ is integration constant corresponding to the bulk Weyl tensor projection ("dark radiation"), and as usual, $\Lambda_4 = \frac{1}{2}\kappa^2(\Lambda + \frac{1}{6}\kappa^2\lambda^2), \quad G_{(4)} = \frac{\kappa^4\lambda}{48\pi}$. The energy density in this model scales as

$$\varepsilon = \lambda\left(\sqrt{(1 + C/a^4} - 1\right),$$

where C is the integration constant. Surprisingly, the constraint equation comes back to that of the YM conformally symmetric cosmology

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_{(4)}}{3}\Lambda_4 + \frac{\mathcal{C}}{a^4} - \frac{k}{a^2},$$

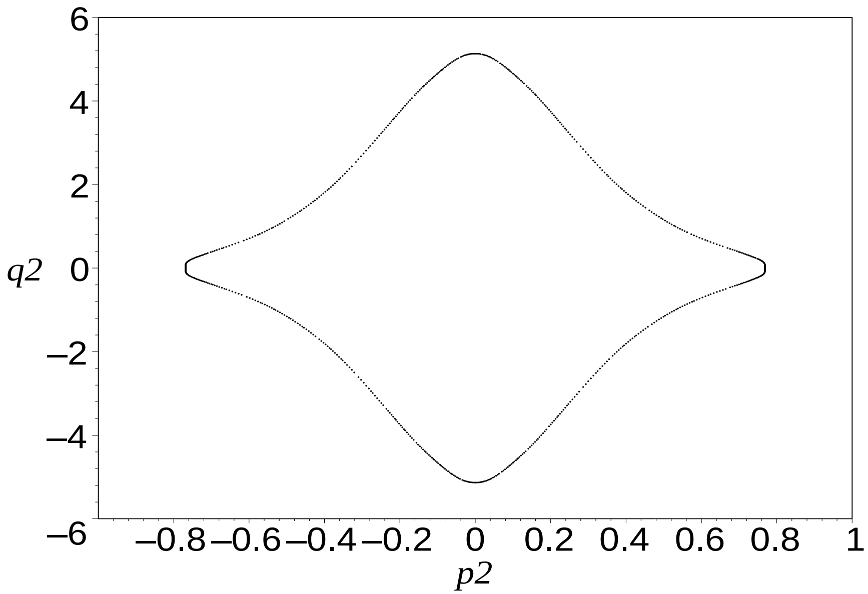
where the constant $\mathcal{C} = \mathcal{E} + \kappa^4\lambda^2 C/36$ includes contributions from both the "dark radiation" and the YM energy density.

NBI and YM chaos

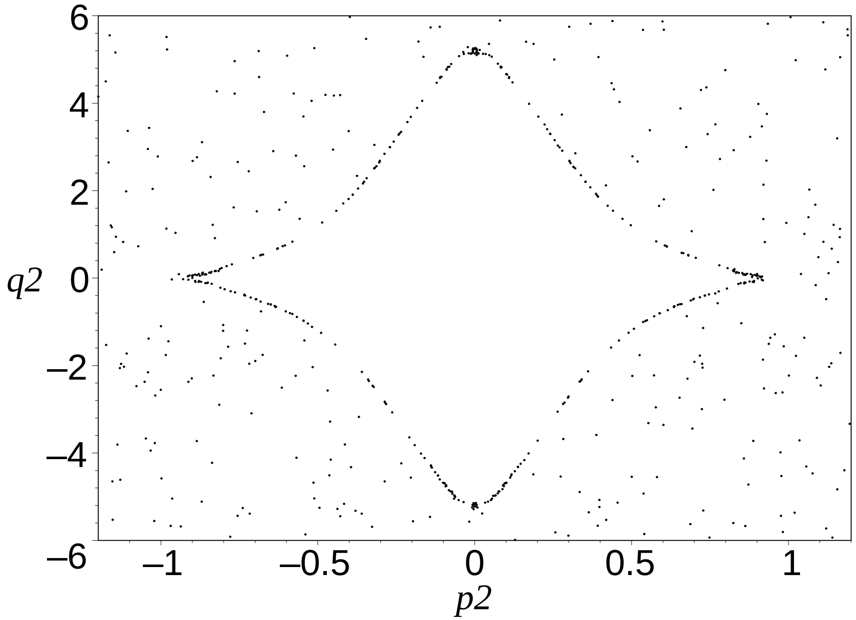
It is well-known that homogeneous YM dynamics is chaotic Saviddy et al). In the isotropic cosmology case, as we have seen, YM dynamics is regular, but once an isotropy is broken, the chaotic behavior is manifest (Barrow, Levin, 97'). It turns out, that in the case of NBI lagrangian, YM chaos is stabilized, which may serve as an evidence for smoothing stringy effect on classical chaos (Gal'tsov, Dyadichev, Moniz 03', 04'). Consider Bianchi I metric

$$ds^2 = N^2 dt^2 - b^2(dx^2 + dy^2) - c^2 dz^2,$$

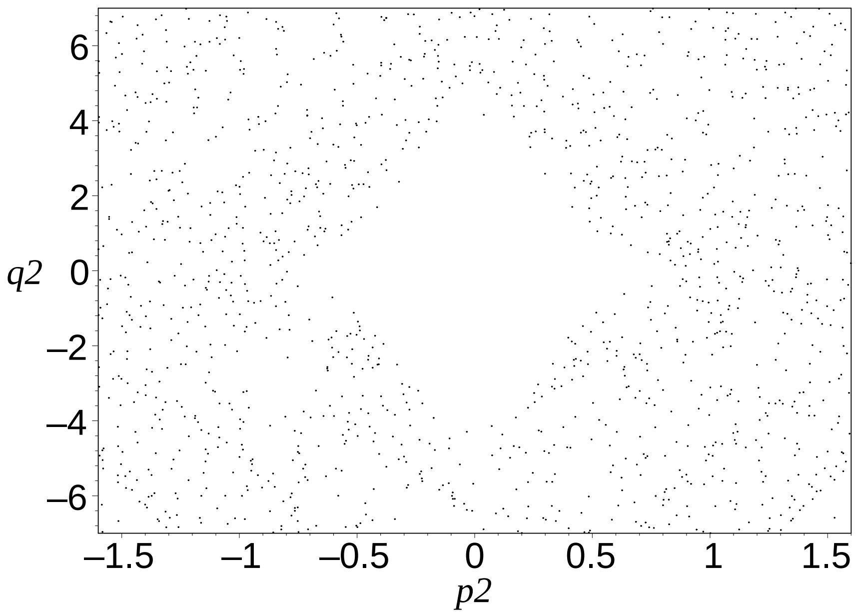
with N, b and c depending on time, and the corresponding YM field parameterized by two functions u, v of time: $A = T_1 u dx + T_2 u dy + T_3 v dz$ The following Poincare sections illustrate chaos-order transition at $\beta = 0.317$ (regular in strong NBI side)



Poincare section for $\beta = 0.31$



$\beta = 0.317$



$\beta = 0.32$

Conformal symmetry breaking and DE

Conformal symmetry breaking in NBI theory demonstrates the occurrence of negative pressure, but its value $P = -\varepsilon/3$ is still insufficient to serve as DE. Meanwhile, a stronger violation of conformal symmetry may provide EOS with $\varepsilon \sim -1$. Such violation can be of different nature

- Quantum corrections
- Non-minimal coupling to gravity
- Dilaton and other coupled scalar fields, including Higgs
- String theory corrections

Assuming the Lagrangian to be an arbitrary function $L(\mathcal{F}, \mathcal{G})$ of invariants

$$\mathcal{F} = -F^a_{\mu\nu} F^{a\mu\nu}/2 \ \text{ and } \ \mathcal{G} = -\tilde{F}^a_{\mu\nu} F^{a\mu\nu}/4,$$

one finds the pressure and the energy density (conformal time)

$$\begin{aligned} P &= L + \left(2\frac{\partial L}{\partial \mathcal{F}}[2(k-h^2)^2) - \dot{h}^2] - 3\frac{\partial L}{\partial \mathcal{G}}\dot{w}(k-h^2) \right) a^{-4} \\ \varepsilon &= -L + \left(6\frac{\partial L}{\partial \mathcal{F}}\dot{h}^2 + 3\frac{\partial L}{\partial \mathcal{F}}\dot{h}(k-h^2) \right) a^{-4} \end{aligned}$$

For a simple estimate consider power-low dependence

$$L \sim \mathcal{F}(\mathcal{F}/\mu^2)^{\nu-1}$$

where μ has dimension of mass. Then in the electric (kinetic) dominance (E) regime one obtains

$$w = \frac{P}{\varepsilon} = \frac{3-2\nu}{3(2\nu-1)}$$

For certain ν this quantity may be arbitrarily close to -1 or even less. A E-phantom regime is possible. In the magnetic (potential) dominance (B) regime $w = 4\nu/3 - 1$ the value -1 can not be reached, but an admissible DE regime is also possible.

Another form of the lagrangian (suggested by quantum corrections) is

$$L \sim \mathcal{F} \ln(\mathcal{F}/\mu^2)$$

Then the energy density is

$$\varepsilon = 3\left(T[\ln(\mathcal{F}/\mu^2)+2] + V\ln(\mathcal{F}/\mu^2)\right) a^{-4}$$

and

$$w = \frac{P}{\varepsilon} = \frac{(T+V)\ln(\mathcal{F}/\mu^2) + 2(2V-T)}{3(T+V)\ln(\mathcal{F}/\mu^2) + 6T}$$

where $T = \dot{h}^2,\ V = (k-h^2)^2$. It is easy to see that $W \sim -1$ for $\ln(\mathcal{F}/\mu^2) \sim -1$. In this case DE regime is transient. The phantom regime is also possible.

Thus, DE conditions for the YM homogeneous and isotropic field can arise indeed as a result of breaking of the conformal symmetry.

Weinberg–Salam cosmology

This next model may be qualified as “realistic”: it describes dynamics of the phase transition with both YM and Higgs homogeneous condensates. The lagrangian includes interacting the SU(2) YM and the complex doublet Higgs:

$$L_m = \frac{1}{2e^2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{2}(D_\mu\Phi)^\dagger D_\mu\Phi - \frac{\lambda}{4}(\Phi^\dagger\Phi - v^2)^2.$$

Complex doublet Higgs (unlike the real triplet) admits homogenous and isotropic configurations, and only in the spatially closed universe, in which case

$$A = e\frac{1-h(t)}{2}UdU^{-1}, \quad \Phi = f(t)e^{i\xi(t)}U\Phi_0, \quad \text{where} \quad U = \exp(ir\boldsymbol{\sigma n}),$$

with three real functions $h(t),\ f(t),\ \xi(t)$. The residual symmetry of this ansatz is $O(1)$ (reflection h to $-h$.) The phase rotation of Higgs $\xi(t)$ can be integrated out (it gives no potential) using the EOM

$$\frac{d}{dt}\left(\frac{f^2a^3\dot{\xi}}{N}\right) = 0 \Rightarrow \dot{\xi} = \frac{\sqrt{2}jN}{f^2a^3}$$

with constant internal momentum number j. The corresponding stress tensor then acquires an additional isospin centrifugal potential term

$$V_j = \frac{j^2}{f^2a^6}.$$

One-dimensional reduced action

The resulting one-dimensional lagrangian contains

- Scalar kinetic and potential terms

$$T = \frac{\dot{f}^2}{2N^2}, \quad V = \frac{\lambda}{4}(f^2-1)^2, \quad V_j = \frac{j^2}{f^2 a^6};$$

- Gauge field kinetic and potential terms

$$K = \frac{3\dot{h}^2}{2N^2 a^2}, \quad W = \frac{3(h^2-1)^2}{2a^4};$$

- Interaction potential

$$V_{int} = \frac{3f^2(h+1)^2}{4a^2}.$$

The interaction term breaks the residual $O(1)$ invariance: the YM $h = \pm 1$ minima are no more equivalent dynamically. The corresponding energy density and pressure read :

$$\varepsilon = T + V + V_j + K + W + V_{int}$$

$$p = T - V + V_j + \frac{K}{3} + \frac{W}{3} - \frac{V_{int}}{3}$$

The Friedman equation for the scale factor reads

$$\frac{\ddot{a}}{a} = -\frac{\kappa}{3}(2T + 2V_j + K - V + W),$$

so cosmic acceleration can be generated only by the Higgs potential term allows the accelerated expansion. Dominance of different potentials gives the following EOS:

V	$w = -1$	Cosmological constant
V_j	$w = 1$	Stiff matter
W	$w = 1/3$	Radiation
V_{int}	$w = -1/3$	String gas

An analytic solution exist when the scalar field is frozen in the local maximum of the Higgs potential and the phase is not rotating :

$$f = 0, \quad j = 0, \quad a^2 = \frac{e^{-\gamma t}}{4\gamma^4 C_2}(4 - \gamma^2\eta^2) + \frac{2}{\gamma^2} + C_2 e^{\gamma t}.$$

with $\gamma^2 = \kappa\lambda/3,\ \eta^2 = 4\kappa C_1/3$, and $C_1,\ C_2$ being the integration constants. Depending on parameters, it may describe both an exponential expansion, or contractions to the singularity. If $\gamma\eta = 2$ one has the static solution with $a_0 = \sqrt{2}/\gamma$.

Dynamical system for $j = 0$

The evolution is described by the six-order dynamical system with stationary points satisfying the equations

$$\frac{3}{2a^2} f(h+1)^2 + \lambda(f^2-1)f = 0,$$

$$\frac{3}{2} f^2(h+1) + \frac{6}{a^2}(h^2-1)h = 0,$$

$$\frac{\lambda}{4} a(f^2-1)^2 - \frac{3}{2a^3}(h^2-1)^2 = 0$$

with the constraint $\kappa a^2 \varepsilon = 3$. The above static solution corresponds to the point $f = h = 0,\ a^4 = 6\lambda$. It described the false vacuum Higgs, the interaction with YM being switched off. Correspondingly, the critical points of YM are the minima $h = \pm 1$ and the maximum $h = 0$ from which only the the last one satisfy the constraint.

For $l = \sqrt{8\lambda/3} > 1$, there is another critical point

$$f^2 = h = \frac{l-1}{l+1}, \qquad a = 2\sqrt{1-w}$$

This is neither false, nor true Higgs vacuum: interaction with YM shifts the critical point to some intermediate value $0 < f_0 < 1$. Similarly, the critical points of YM are shifted to $h_\pm = (1 \pm \sqrt{1 - a^2 f_0^2})/2$. From the constraint equation it follows that for $l < 3$ one has a local maximum, for $l = 3$ maximum and minimum coincide, for $l > 3$ there is a local minimum.

Eingenvalues

Linearisation around the above two critical points give the stability matrices

$$M_1 = \begin{pmatrix} \frac{3l(l-1)}{8} & 0 & 0 \\ 0 & \frac{3l}{2} & 0 \\ 0 & 0 & \frac{l}{2}, \end{pmatrix}, \quad M_2 = \begin{pmatrix} -\frac{3l^2(l-1)}{4(l+1)} & -\frac{3l\sqrt{l-1}}{4\sqrt{l+1}} & \frac{3l^2\sqrt{l-1}}{2\sqrt{2}(l+1)} \\ -\frac{6l\sqrt{l^2-1}}{(l+1)^2} & \frac{3l(3-l)}{2(l+1)} & -\frac{3\sqrt{2}l(l-1)}{2(l+1)^{3/2}} \\ -\sqrt{\frac{l-1}{2}} & \frac{\sqrt{2(l+1)}(l-1)}{4l} & \frac{1}{2} \end{pmatrix}.$$

The eigenvalue of $\det(M_1 - \mu^2 I) = 0$ for Higgs is $\mu_f^2 = 3l(l-1)/8$, for $l > 1$ one has a node, for $l < 1$ a focus. This means that $V_{\rm int}$ supersedes V, so the effective potential has minimum for $f = 0$ instead of maximum. For YM we obtain $\mu_f^2 = 3l/2 > 0$, hence a node at $h = 0$. For the scale factor $\mu_a^2 = l/2 > 0$, also a node describing exponential expansion and/or contraction. Critical value a_0 corresponds to compensation of inflationary Higgs potential V and deflationary YM potential W, so if start with $a > a_0$, a new accelerated expansion period begins, if $a < a_0$ it will collapse.

For M_2 one has a cubic equation for μ^2. One root is always real and negative (focus). Two others are complex on the interval $l \in [1.8,\ 7.5]$, with equal real parts and imaginary differing by sign, i.e. four spiral orbits from one focus to another differing by directions and orientation (left/right winding). For $1 < l < 1.8$ roots are real and positive (nodes), while for $l > 7.5$ negative (elliptic orbits). Thus for $l > 7.5$ one has foci in the full six-dimensional phase space, hence the system is stable.

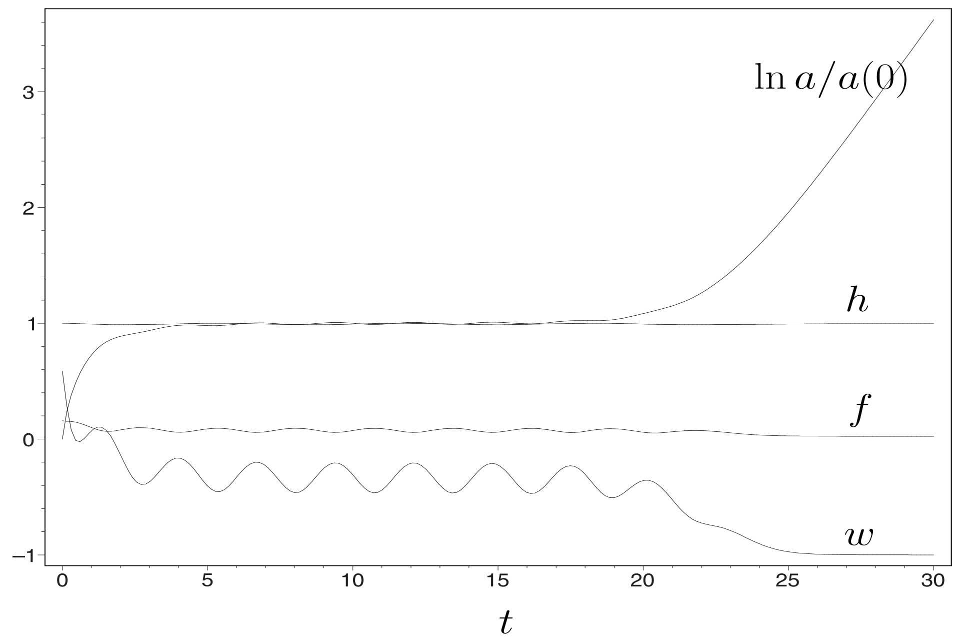
Metastable $w = -1/3$ state

Physically, the motion near the stable critical points corresponds to the EOS $w = -1/3$ (string gas). The system can spend rather long time in such a state until deviations from linearity become significant. Then it exhibits transition either to inflation with $w \sim -1$ or to a dust-like state $w \sim 0$. The value of the Hubble parameter of inflation depends on the value of Higgs in the metastable phase and it is not defined by the full height of the Higgs potential like in the usual inflaton picture. In particular, on the lower bound $l = 3$ for existence of the metastable string phase (YM field near minimum of the potential) we get $f_0^{min} = 1/\sqrt{2}$, hence a weaker inflation. For zero isospin angular momentum j the value of the resulting Hubble constant of inflation decreases with increasing duration of the metastable phase. The scale factor during the string gas era exhibits small oscillations around the value corresponding to the static Einstein universe.

The relative value of the isospin centrifugal potential is weighted by the isospin momentum j. With growing j duration of the metastable phase increases, and the Hubble parameter of inflation grows up, since the system approach the authentic false vacuum. For large enough j the Hubble constant of inflation practically does not depend on the duration of quasistatic universe.

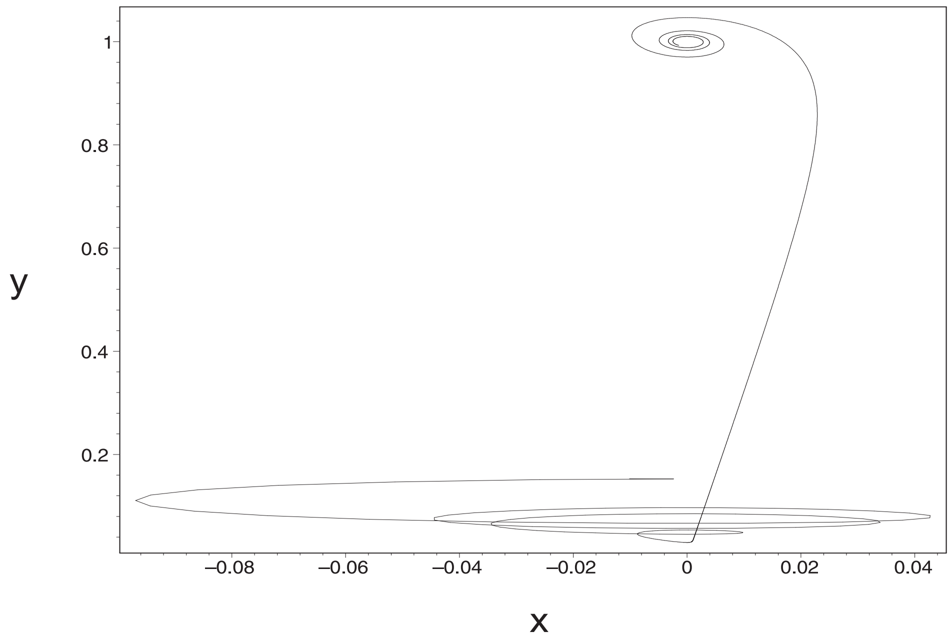
Numerical experiments

Numerical experiments confirm expectations that the system exhibits non-thermal phase transition from the state of usual matter $w > 0$ to inflation $w \sim -1$ through the metastable stage with the EOS of string gas $w \sim -1/3$. This plot shows evolution of YM-Higgs-gravity coupled system starting from some state $w > 0$ with l=0.3 (lambda=0.033), j=0.037. This is followed by long stage of quasistable oscillations of w around the value $w \sim -1/3$ after which the system starts to inflate. The slope of the logarithm of the scale factor is an effective Hubble constant, its value depends on duration of the metastable phase: typically the longer is metastable phase, the smaller is Hubble parameter of inflation. After a finite time of inflation the system returns to $w > 0$ state (not shown).



Evolution of Higgs

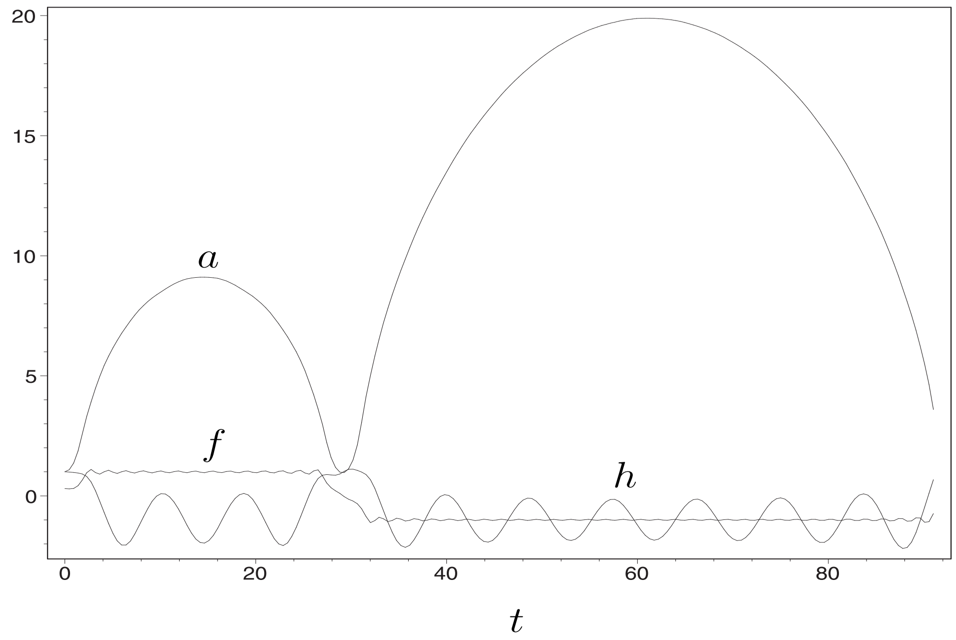
During the metastable $w = -1/3$ stage Higgs oscillates with small amplitude till transition to inflationary phase when it is stabilized on some non-zero value. The accelerated expansion at the this stage is due to interaction between the gauge and scalar fields which dynamically 'raises up' the Higgs field minima. Inflation end when Higgs reaches the true vacuum $f = 1$ after some oscillations.



During the inflationary stage the interaction term is suppressed like a^{-2} and can be neglected.

Time-developed Multiverse

Inflation period stops when Higgs reaches the true vacuum, as usual. Further evolution corresponds to EOS with $0 < w < 1$ and leads to contraction either to singularity, or to bounce, after which the new cycle of expansion begins



New initial conditions depend on the state of YM components, which oscillate rather independently. As a result, each new cycle starts with different (chaotically distributed) initial conditions. The overal picture can be viewed as Multiverse developed in time.

Concluding remarks

- SU(2) YM triplet perfectly fits to homogeneous and isotropic cosmological metrics. The same for complex doublet Higgs (but not for real triplet)

- Including YM condensates into cosmological scenarios leads to novel features of cosmological phase transitions, not fully explored yet

- Both string (Born-Infeld) and standard model (Weinberg-Salam) motivated scenarios demonstrate special role of the EOS $w = -1/3$ corresponding to zero acceleration. In particular, WS model predicts possibility of long metastable phase of almost zero acceleration before inflation

- Global evolution exhibits cyclic structure with non-identical cycles whose parameters are chaotically distributed.