Localization in QFT

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Abstract

Collection of problems on short-distance QCD, based mostly on the results of ITEP lattice group

Notion of localization

cond-matter (Anderson)

Eigenfunction $\Psi_n(x)$

$$\int |\Psi_n(x)|^2 d^4x \equiv 1$$

$$(IPR)_n \equiv V_{tot} \int |\Psi_n(x)|^4 d^4x$$

$IPR \sim 1$ plane wave

$$IRR \sim \frac{V_{tot}}{V_{loc}}$$

Mobility edge

Plane waves are solutions for ideal crystal

Impurities induce

- localization for low-lying states

$$V_{loc} \ll V_{tot}$$

- mobility edge λ_{edge} for $\lambda_n > \lambda_{edge}$ no localization

$\frac{1}{2}\hbar\omega$ as "impurity"

QCD vacuum is studied through "configurations"

$$\{A_\mu(x_n)\}$$

$$|A_\mu(x)| \sim \frac{1}{a}$$

a - lattice spacing

One might think that measurement itself induces impurities

Scalars in the background

$$\mathcal{D}^2 \varphi_n = \lambda_n \varphi_n$$

solved numerically for a given $\{A_\mu(x_n)\}$ and different color spin of scalars φ

Both localization and mobility edge are found

Simple-to-remember difficult-to-explain scaling laws

Dependence on color spin

$T = 1/2$ (color)

$V_{loc} \sim \Lambda^4_{QCD}$

$\lambda_{edge} \sim \Lambda^2_{QCD}$

$T = 1$

$V_{loc} \sim \Lambda^2_{QCD} \cdot a^2$

$\lambda_{edge} \sim \Lambda_{QCD} / a$

$T = 3/2$

$V_{loc} \sim a^4$

$\lambda_{edge} \sim a^{-2}$

Conclusion to part I

At least naively, perturbation theory never works for adjoint scalars (even after mass renorm.)

Greensite, Olejnik, + ITEP

Constraints on "impurities"

Zero-point fluctuations are part of theory

As a result, some matrix elements are to be stable against

$a \to 0$

The known examples refer to fermions (chirality protection)

Fermionic topological modes

Two famous relations

- $\langle Q_{top}^2 \rangle \sim \Lambda_{QCD}^4 V_{tot}$

(Veneziano - Witten)

$$Q_{top} = n_+ - n_-$$

- $\langle \bar{q} q \rangle \sim \rho(\lambda_n \to 0)$

(Banks - Casher)

Both should be independent on a and they are independent

Topological volume shrinks to zero:

For zero-modes

$$V_{loc} \sim a^2 \cdot \Lambda_{QCD}^2$$

For near-zero modes

$$V_{loc} \sim a \cdot \Lambda_{QCD}^3$$

(Ilgenfritz et al 0912...)

Conclusions to part II

"Instanton size"

depends on resolution of measurements which is the lattice spacing. It vanishes in the continuum limit of $a \to 0$

Solutions in dual language

In direct formulation of QCD instantons are the only non-pert. solutions. They get localized.

In the dual formulation of YM which is presumed IR completion of QCD there are further solutions. Are they localized?

Magnetic strings

Typically, there are compact extra dimensions and corresponding new topological solutions (S-S model)

Same typically, there are D2 Branes which are 2d defects in 4d with non-trivial θ-dependence. Classically tension disappears on the horizon

Localized image

Infinitely thin,
infinitely hot surfaces

$$(\text{Area}) = c\, \Lambda_{QCD}^2\, V_{tot}$$

$$(\text{Action}) = c' \, \frac{(\text{Area})}{a^2}$$

with θ-dependence
(as expected)

and scalars living on
the surfaces
(not expected)

Scalars

Remarkable properties:

$$\langle 0|\varphi|^2 0\rangle = c_3 \Lambda^2_{QCD}$$

Mass is fine tuned

$$m^2_{phys} = \frac{const}{a}\left(M(a) - \frac{\text{"}\ln 7\text{"}}{a}\right)$$

where classical action

$$S_{cl} = M(a)\cdot L$$

On the lattice

$$M(a) \approx \frac{\ln 7'}{a}$$

Conclusion on part III

Accumulating evidence that localized images of solutions to dual formulation are seen

Overall conclusions

Non-trivial "indices" in

$$(\Lambda_{QCD} \cdot a) \equiv \epsilon$$

$$V^{sc}_{loc} \sim \epsilon^0, \epsilon^2, \epsilon^4$$

$$V^{ferm}_{loc} \sim \epsilon^1, \epsilon^2$$

$$V^{surf}_{loc} \sim \epsilon^2$$

$$\lambda^{sc}_{edge} \sim \epsilon^0, \epsilon^{-1}, \epsilon^{-2}$$

$$\lambda^{surf} \sim \epsilon^{-2}$$

+ CONFIRMATION OF REALITY OF SOLUTIONS TO DUAL FORMULATION